GEMS Challenges ----- Mobius Bands

SUBJECT	**Suggested grades**	**Cautions / concerns**
Mathematics--topology	3-8	

The Challenge—How can a piece of paper have only one side?

	LEADER ACTIONS	GIRL ACTIONS
INTRODUCTION	 Take the ball of playdough out of the can, knead it and form it into a mug—do not tear away any clay—just mold the ball into a mug shape with a handle. Ask the girls how many holes the mug has. (Answer—one) Carefully in full view of the girls, compress the mug into a donut shape, again, not tearing any clay but just shaping it. Ask the girls how many holes in the doughnut. (Answer—one) Ask them—are the mug and the doughnut the same? (topologically, yes)	Look at the mug—how many holes does it have? Defend your answer.
BACKGROUND INFORMATION	We are investigating the mathematical topic of topology—the properties of objects that are preserved even though they are twisted, stretched, and deformed.	
ACTIVITY	Hold up a strip of precut paper. Ask how many sides it has. (Two) Take this strip and tape the ends together, making a loop. Pass it around, asking how many sides it has. Demonstrate that is has two sides by drawing a line down the middle, emphasizing that you have to pick up the marker and move it to draw the line on the other, or inside. Get agreement that this loop has two sides. Give each girl a strip of paper. Have her	Watch and answer. Make a loop by taping the ends together and

	LEADER ACTIONS	GIRL ACTIONS
	make a loop by taping, and drawing to confirm that her loop has two sides. Now, ask the girls to watch you while you take a new strip of paper, twist it once and tape the ends together to make a loop. Ask them how many sides it has now. Remind them that we agreed that the marker test proves how many sides. With help from a student, draw a line beginning at the tape and show that your line ends up at the beginning without lifting your marker. How many sides does that strip of paper have? Give each girl a few strips and let her make her own Mobius strip. Do the drawing test on each loop. Discuss the conclusions. Now, ask the girls what will happen if you cut the loops—the single loop with two sides and the Mobius strip with one? Predict and discuss. How many sides are there now?	drawing a line down the middle to test and prove that it has two sides. Observe and guess with the twisted loop. Test your own strips. How many sides do they have? What do you think is happening? Cut your loops on the lines
CONCLUSION	How is this related to the mug and doughnut? How can this relate to the real world?	Brainstorm ideas
REFLECTION	Ask two girls to take a card from the reflection cards and share their experiences.	

Supplies per table

New container of playdough

Strips of paper at least 1 inch by 14

Scissors and markers

Tape

Preparation needed

- Try this first so you are comfortable

Comments

There is never enough time, so encourage the girls to try variations at home.

It is often said that idea of evolution to biology is same as the ideas of topology to mathematics. Topology refers to the relationship between spatial features or objects. In terms of functionality, topology is important in (at least) three important ways:

First, topology is necessary for certain spatial functions such as network routing through linear networks. Here the idea is that if line features do not share common nodes, that routes cannot be established through the network.

Second, topology can be used to create datasets with better quality control and greater data integrity. Topology rules can be created so that edits made to a dataset can be 'validated' and show errors in that dataset. An example would be the creation of a new manhole/sewer access feature outside a polygon dataset of road features.

Third, by creating topological relationships between feature classes, features can be shared across feature classes. In other words, if you open one dataset and edit/move a line feature that is shared between two feature classes, then both feature classes will be updated to reflect the edits. This is massively helpful for keeping datasets synchronized. An example would be a river feature that defines a administrative boundary (where the river moves over time), or the boundary of a municipal area and zoning polygons.

Topological spaces show up naturally in almost every branch of mathematics. This has made topology one of the great unifying ideas of mathematics.

Topology can be thought of as abstracting geometry by removing the concept of distance. In general, abstractions are useful because they allow you to compare two things that only differ in properties that are irrelevant to you and can be seen to be equivalent once you remove those properties from the discussion. Consider, for instance, a cage. If your only purpose in constructing a cage is to keep something contained, then what matters is whether or not the cage is closed, not its actual shape. A cup (of any shape) can be used to cage a fly; a toilet paper roll cannot.

Roller Coasters

Six Flags Magic Mountain recently remodeled Colossus. Part of this remodeling involved changing the track layout from a racing layout to a Möbius layout[1]. In the previous layout, the ride was composed of two separate tracks placed side by side. In the current layout, the track starting in the first station ends in the second station and the track starting in the second station ends in the first station, giving one continuous track instead of two separate tracks. Now, I don’t know much about building roller coasters in real life, but I did used to play a lot of Roller Coaster Tycoon and when building a dual track rollercoaster, the biggest consideration in choosing the station topology was that using the Möbius layout gave you a single ride while using separate tracks gave you two rides which affected the official number of roller coasters in your park (which I assume was a big part in the park rating, park value, and guest generation algorithms), the roller coaster’s rating, guest thoughts, etc.

Public Transportation

When designing and building public transportation systems, the network topology of the system is analyzed separately from the geometry. The topology gives the general structure of the system while the geometry, i.e. the exact shape of the transportation lines, where to put the stops, etc. deals with the specifics (to use an analogy, choosing the topology is like choosing between Mac and Windows and optimizing the geometry is like changing the settings and appearance to suit your needs). Here in Los Angeles, Metro is currently working on a project that will actually change the network topology of the train system. Most of the tracks will remain the same, but by reconnecting the tracks (the north/south portion of the gold line will become a part of the blue line and the east/west portion of the gold line will become a part of the Expo line), the topology will be changed from X-shape to complex grid.