GEMS Challenges ---- Topology

SUBJECT	**Suggested grades**	**Cautions/concerns**
Math--Topology	4-8	

The Challenge—Begin to understand topology and its uses in math and computer science

	LEADER ACTIONS	GIRL ACTIONS
INTRODUCTION	Present the challenge—Can you take a walk through the famous Russian town of Konigsberg, visiting each part of town and crossing each bridge only once?	
BACKGROUND INFORMATION	This was a famous mathematics problem which puzzled people for years until it was solved and proved by Euler. Königsberg was set on both sides of the Pregel River, and included two large islands - Kneiphof and Lomse - which were connected to each other, or to the two mainland portions of the city, by seven bridges. People tried to devise a walk through the city that would cross each of those bridges once and only once.	
ACTIVITY	Pass out the bridge handouts and show the original 'map', pointing out the river, bridges and the islands. Show the next picture, the map as a drawing, and then the map as a simplified figure. Make sure the girls understand how the map was simplified into the drawing and that the capital letters A-D represent land/islands, and the lower-case letters p-v represent the bridges. Show how to try the problem by drawing over the lines. Start the challenge—can they draw each line p-v only once without removing the pencil? They can start at any point. This cannot be done. Turn the paper over and have them try	Review the drawings and maps. Ask questions if they don't understand. Try it many times. What is happening? Try the simpler figures on

	LEADER ACTIONS	GIRL ACTIONS
	the simpler shapes, starting again at any point and remembering these rules: Draw all the lines, but never go over any line more than once and don't lift the pencil from the paper. Fill out the grid. Ask them what they notice? How does this relate to the original bridge problem?	the back of the page. Fill out the grid with your results. What do you notice?
CONCLUSION	Use the Simplified Konigsberg Answer to discuss the answer. There may be some hot discussion here. Ask them: Do you agree? How do you think this kind of problem relates to the real world?	Talk about the answer
REFLECTION	Ask 2-3 girls to draw a card and reflect on this experience	Reflect on this activity.

Supplies

Worksheet—Konigsberg Bridge Figures—3-4 copies for each girl

Pencils

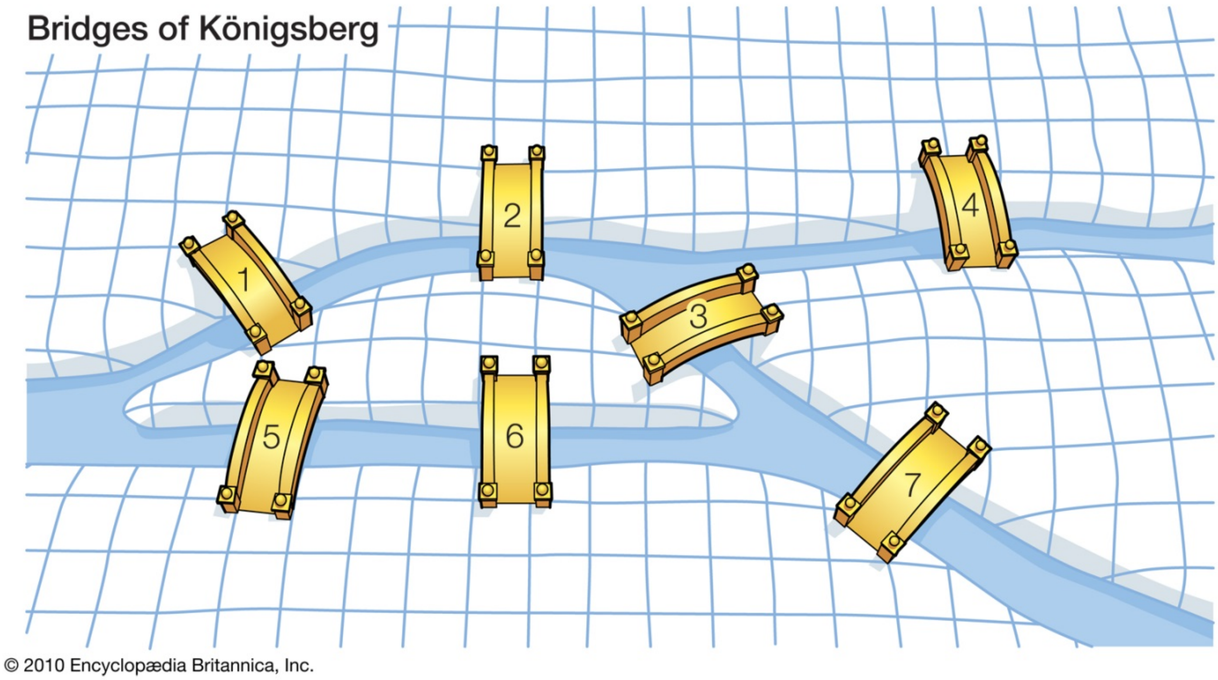
Preparation needed

- Try this at home first
- Read the Math is Fun paper to be sure you understand this.

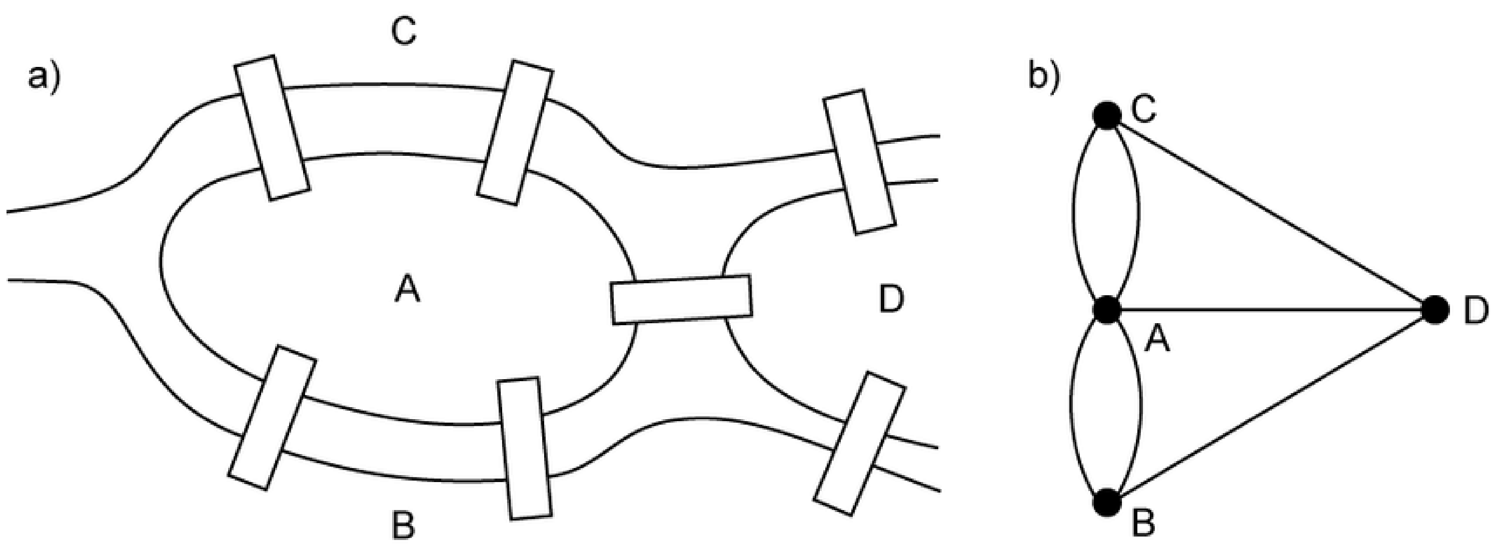
Comments

Girls will not see this at first. There will be a lot of experimentation, and many questions. Be ready.

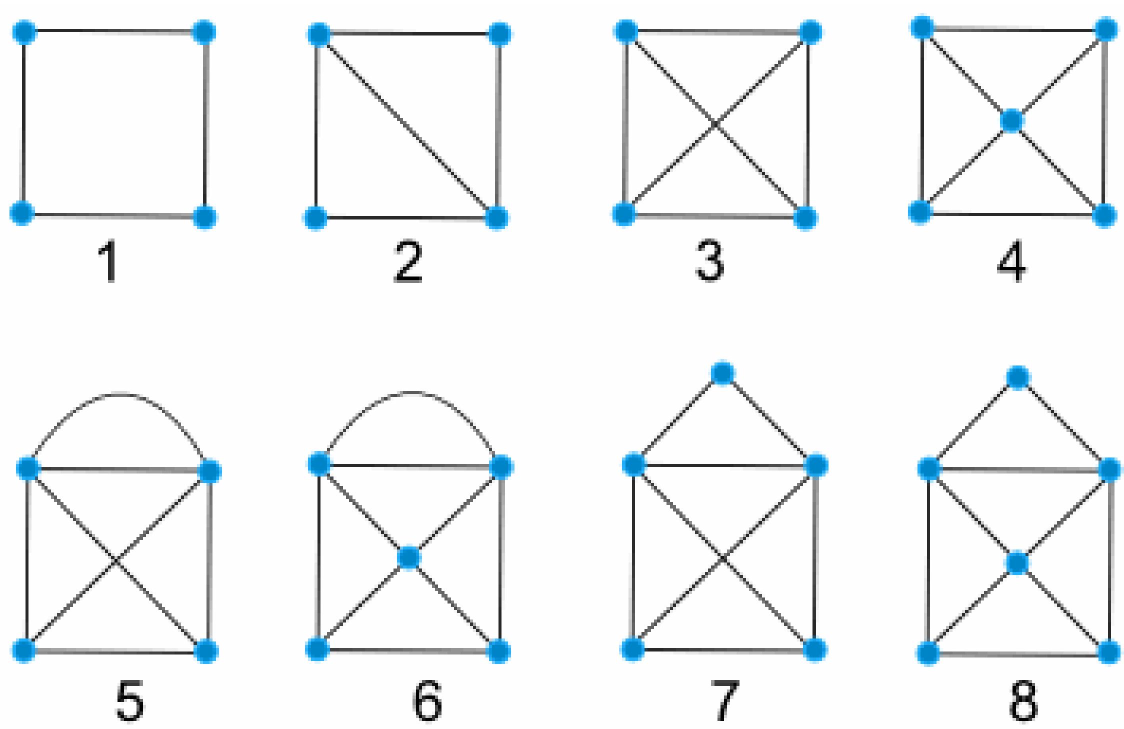
The Konigsberg Bridge Problem



The original 'map' of the bridges



The 'map' as a drawing—ready to be solved.



Shape	Success?

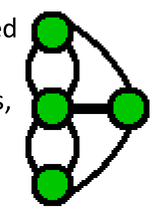
Konigsberg simplified answer

From https://web.archive.org/web/20070118135955/http://www.jimloy.com/puzz/konigs.htm

Look at the diagram of the city of Königsberg (now called Kaliningrad). It is on both banks of the river Pregel, as well as on two islands. And it had seven bridges, as shown. Some people speculated that there might be some path through the city, which would cross all seven bridges only once. People tried this, but never succeeded. Leonhard Euler (pronounced "oiler") showed that it was impossible, and thus created Graph Theory.

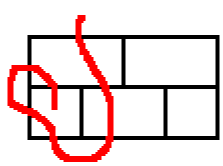
The proof that the path is impossible goes like this. Look at the island on the right. There are three bridges connecting this island to elsewhere. If our path begins on this island, then it leaves the island, comes back, and then leaves the island, using all three bridges in some order. If our path starts elsewhere, it eventually comes to this island, leaves it, and then comes back.

We have shown that our path either starts here or ends here. One of the endpoints of our path is on this island. The same holds for the northern shore and the southern shore. One of the endpoints must be in the northern part of the city, and one endpoint must be in the southern part of the city. Our path cannot have three endpoints. So the path is impossible. In fact we can show that the left island must also contain one of the endpoints. Any area connected to an odd number of bridges must contain one of the endpoints.



The picture of Königsberg is equivalent to the graph on the right. The bridges are represented by the edges of the graph. Such a graph, and the above observation that any vertex (area of the map) connected to an odd number of edges (bridges) must contain one of the endpoints, simplify the solution to many puzzles.

More information:



Here is another puzzle. We have the black boxes on the left. Our task is to draw a path (the one in red is just beginning) so that it goes through each wall only once. Each wall is the black line between two wall intersections. The upper right box, therefore, has five walls which must be crossed. Our path cannot go through an intersection. This is very similar to the Königsberg puzzle. The walls are exactly like the bridges. In this puzzle, three of the areas (vertices) have an odd number of walls (bridges or edges). And so our path must have three endpoints, and is impossible. Two of the vertices have an even number of edges, and are no problem. If the puzzle had had a solution, i.e. if the path has two or fewer endpoints, the graph would tell us where those endpoints must be, and the solution would become simple. If the graph specifies fewer than two endpoints, then we can put the unspecified endpoints anywhere we want.