A NARRATIVE INQUIRY OF FEMALE MATHEMATICS/STEM EDUCATORS: CROSSING BOUNDARIES AMONG MULTIPLE CONTEXTS

by

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ABSTRACT

The limited numbers of women in advanced mathematics courses is a critical factor hindering women's academic and professional access to science, technology, engineering, and mathematics (STEM) fields. Informal learning environments have the potential to play a significant role in promoting the participation of girls and women in mathematics/STEM fields. However, research that addresses the intersection of informal education, mathematics education, and women's studies is minimal. Specifically, little is known about informal educators' lived experiences in facilitating girls' learning. Based on four years of working alongside Laura, the founder of Girls Excelling in Math and Science (GEMS) clubs, I conducted a narrative inquiry that explored our boundary crossing experiences as we engaged in a GEMS collaboration. The exploration focused on Laura's narratives of her past, present, and future experiences that shape her identity as an informal educator. During the exploration of Laura's experiences, I reflected on and inquired about my own personal and professional experiences across multiple contexts that inform my evolving identity as an educator. The theoretical framework of this study is informed by feminist theory and boundary-crossing perspectives. Feminist theory guides me to perceive our narrative of experiences from a women's perspective while the boundary-crossing framework provides an analytic lens to understand our interpersonal and intrapersonal boundary crossing experiences. Because of the nature of the narrative inquiry, data were co-constructed between Laura and me in various forms: interviews, field notes, family stories, autobiographical writing, documents, conversations, emails, etc. I employed Polkinghorne's (1995) *narrative analysis* and *analysis of narrative* approaches to analyze data. First, I utilized a *narrative analysis* approach to generate three holistic plots: (1) narratives of becoming female educators, (2) boundary-crossing collaboration in the midst of GEMS, and (3) conceptualizing mathematics across multiple contexts. An *analysis of narrative* approach was used to generate themes that unfold the meanings of stories, moments, and events and configure the plot. In the findings, I portrayed the three plots which allowed me to rediscover and reconstruct our personal practical knowledge across the contexts. Building on the findings, I discuss how female educators' narratives of experiences inform their personal practical knowledge, which empowers girls' and women's personal and social experiences in mathematics/STEM. Laura and I cross multiple boundaries engaging in collaboration which provides an example of the boundary crossing collaboration between mathematics education and informal education. Based

on the findings, I describe how informal learning STEM environments provide potential spaces to implement alternative curricula to humanize mathematics. Two evolving mathematics-related tasks illustrate our experiences of humanizing mathematics in GEMS. This study is situated at the intersection of mathematics education, informal education, and women's studies, which significantly impacted Laura, myself, and GEMS, the context in which this study took place. This study provides an example of the possibilities of building boundary-crossing collaborations between the mathematics education community and the informal education community to empower girls and women in mathematics/STEM. Drawing on this dissertation study, one future research direction focuses on implementing and further developing humanized mathematics curricula in informal learning environments. Another research direction is using intersectional feminist theory to understand women's differences regarding multiple social constructs (e.g., race, gender, class, ethnicity) to explicate the dimensions of inequality women face in mathematics/STEM. The study also suggests future practical work for mathematics education to foster alternative ways of conceptualizing mathematics regarding curriculum and approach. Mathematics educators could contribute to creating a learning community and providing professional development opportunities to support informal educators.

CHAPTER 1. INTRODUCTION

In the United States, women are generally underrepresented in science, technology, engineering, and mathematics (STEM) fields, but most underrepresented in mathematics-intensive STEM disciplines (e.g., Engineering, Physics) (Ceci et al., 2014). Researchers have suggested that mathematics may be an area that girls and women intentionally avoid when making career decisions (Eccles, 1994; Eccles & Wang, 2016). Thus, mathematics can be a gatekeeper that filters capable women out of advanced study and better-paid occupations (e.g., STEM professions), which continues the reproduction of gender inequality in society (e.g., Ceci et al., 2009, 2014; Leder, 2019; Wang, 2012; Watt et al., 2017). According to the National Center of Educational Statistics (2019), in 2015-2016, only 36% of bachelor's degrees were awarded to females versus 64% to males in overall STEM fields. Women earned less than 43% of doctoral degrees in mathematics and statistics and 19%, 21%, and 19% of doctoral degrees in computer sciences, engineering, and physical science. With the recent increased attention to STEM, promoting women's participation in mathematics is essential. Note that, in this study, the terms *women* or *females* are used to refer to all individuals who identify themselves as women, which may not accord with their sex assigned at birth.

Statement of the Problem

Since the 1970s, a great deal of interest has been given to gender and mathematics. Many studies have focused on the differences in the mathematical performance and dispositions between boys and girls (e.g., Fennema & Sherman, 1978; Hyde et al., 1990; Lindberg et al., 2010). The findings of gender differences in mathematics inspired researchers to investigate the causes of less engagement in mathematics by girls and women. Some researchers have searched for the origins of gender differences in decision-making (i.e., choosing mathematics courses or not) (Eccles, 1994; Eccles & Wang, 2016; Eagly & Wood, 1999) and found that gender differences in mathematics are more related to sociocultural influences than biological factors. Researchers focusing on the learning environments (e.g., Boaler, 2002) have noted that the disaffection of girls to mathematics might be considered a response to particular learning environments. Stereotypical characterizations about girls and women's inferiority in mathematics embodied in macro- and micro-environments also play a part in shaping individual beliefs and motivation over time (e.g., Lubienski & Ganley, 2017; Mendick, 2005).

Although research on gender is prevalent in the mainstream mathematics education literature, most of these studies focus on the achievement gap, paying less attention to girls and women's lived experiences with mathematics (e.g., Damarin, 2008; Lubienski & Bowen, 2000). Gutiérrez (2009) argued that focusing on achievement gaps is inadequate for promoting social justice and that researchers should highlight education over schooling and develop a counternarrative about who contributes to mathematics. In mathematics education research, *women and mathematics* are often treated as equivalent to *gender and mathematics*; in other words, feminist analysis and scholarship are largely absent from discussions of women and mathematics (Damarin, 2008). Therefore, a critical need exists for a feminist perspective in mathematics education research that brings *women* into the center of inquiry.

Researchers have been searching for ways to support girls and women's engagement in mathematics and suggested that the gender disparity in mathematics may result from students' school mathematical experiences (e.g., Boaler, 2002; Cooper, 2011). As girls and women engage in informal mathematics/STEM learning, their interest, motivation, and persistence have been shown to increase while susceptibility to the stereotype threats has been shown to decrease (e.g., Chen et al., 2011; Dasgupta & Stout, 2014; Holmes et al., 2012; Morris et al., 2019). However, the voices and contributions of informal educators, who facilitate this informal mathematics/STEM learning, are mostly absent in mathematics education research (Nemirovsky et al., 2017). As a result, little is known about informal educators' experiences designing and implementing curricula or how they conceptualize mathematics.

Purpose of the Study

Girls and women's experiences and insights on mathematics and its teaching and learning require a closer attention. However, few studies have been conducted to investigate female educators' experiences of crossing the boundaries between formal and informal learning environments and boundaries between individual disciplines and integrated STEM. In addition, in extant literature mathematics is often discussed as a gatekeeper of access to further education in STEM fields, rather than understood in a connected way with learners' social constructions and integrated STEM practices. Note that, according to Oxford English Dictionary, in this study, the term *educator* refers to a person who educates, trains, or instructs which distinguishes from the conventional term *teacher* referring a person who teaches in a classroom or school setting.

In this dissertation, I investigate Laura's and my lived experiences with mathematics crossing multiple boundaries (e.g., culture, language, and discipline) to bridge this gap. Laura is the founder of a girls-only afterschool STEM club–Girls Excelling in Math and Science (GEMS). For the past 28 years, since starting the first GEMS club in 1994, she has devoted herself to promoting girls' participation in STEM. Since Fall 2018, I have collaborated with Laura on the GEMS program to further develop the GEMS program and conduct research with the original GEMS girls (OGGs) who participated in the first GEMS club 28 years ago. Coming to this study in the tradition of narrative inquiry, I position myself as a researcher and a participant. Thus, during the exploration of Laura's experiences, I have been reflecting on and enquiring about my own mathematics experiences across multiple contexts. Specifically, I followed Clandinin and Connelly's (2000) guidance that I work with Laura by "com[ing] to experience not only what can be seen and talked about directly but also the things not said and not done that shape the narrative structure of their observations and their talking" (p. 68).

In this inquiry, exploring personal narratives with a focus on being women and educators helps provide an alternative way of valuing individuals' personal and professional experiences as a mode of knowing. I explored Laura's life histories and her multiple personal and professional identities to understand why she has dedicated her time and efforts to developing and nurturing GEMS. Retelling her personal and professional narratives allows Laura to reconstruct the stories of her experiences and rediscover how these experiences informed her knowledge as a female informal educator. I also intended to capture our evolving personal practical knowledge regarding the nature of mathematics in general and mathematics in GEMS during our cross-boundaries collaboration. I traced back my own personal histories to understand the new and evolving identity as an informal educator. Our personal narratives are constructs within social contexts that mediate self and educational contexts and inform professional identity as educators. Specifically, through investigating Laura's lived experiences, reflecting on my own experiences, and looking across our stories, this study moves female informal educators' voices from the margins of academic dialogue to the center while "developing more nuanced understandings of the multiple power relations that shape women's situations" (McCann & Kim, 2013, p. 25).

In this study, I provide insights into what constitutes empowering mathematics/STEM learning environments that support girls to be active participants in mathematics/STEM with positive mathematical dispositions. Specifically, the following research questions guided this study:

1. How do two female educators' experiences and histories inform their personal practical knowledge to empower girls and women in mathematics/STEM?
2. What are the boundary crossing experiences as two female educators engaging in Girls Excelling in Math and Science (GEMS) collaborate?
3. What role, if any, do two female educators' mathematical experiences play to provide high-quality informal STEM environments for girls?

Significance of the Study

Informal mathematics education is an emerging field that is often excluded from mathematics education research. The extant literature has not provided descriptions of the work of informal mathematics educators and the concurrent absence of outlines for career paths in the informal education field (Nemirovsky et al., 2017). This study is an explicit response to Nemirovsky et al.'s (2017) call for researchers to "consider the contribution, knowledge, and experiences of a larger segment of the population, in particular, those whose voices and practices have traditionally not been represented in mathematics education (formal and informal)" (p. 976). In this inquiry, I also respond to the divide between mathematics and women's studies, which is "substantial and rarely crossed" (Damarin, 2008, p. 101). Therefore, this study will expand the literature related to the intersection of women's studies, mathematics education, and informal education by carefully analyzing the lives, experiences, and interactions of two female educators.

The experiences of female educators devoted to improving the participation of girls and women in mathematics/STEM require closer attention. Their perspectives on what counts as mathematics and who can be successful in mathematics contribute to views of mathematics and its teaching that have the potential to support girls and women in mathematics/STEM. Representation of these women's insights, beliefs, and knowledge contributes to gender equity by creating a basis for foregrounding voices of women who engage in important mathematical work (Damarin, 2008). Through a feminist lens, focusing on women's experiences and analyzing women's storytelling related to mathematics will both build on previous research and fill a knowledge gap in women's and mathematics education studies. The findings enhance girls' and women's mathematical experiences and give "voice" to their viewpoints.

This study also provides a reference for building boundary crossing collaboration between the mathematics education community and communities outside of mathematics education. Efforts taken to cross boundaries between mathematics education and informal education in this study build bridges between school mathematics and informal mathematics, providing opportunities to reframe and humanize mathematics. In particular, informal mathematics learning is predominantly experiential, with the potential to foster alternative ways of conceptualizing mathematics and mediate the traditional image of mathematics. The two presented mathematics tasks illuminate alternative ways of conceptualizing mathematics generated from the boundary crossing collaboration.

In future iterations of this research, the mathematics education field will have the tools and acknowledgment needed for drawing on educators' personal and professional experience with mathematics. Other STEM disciplines might also benefit from the insights on crossing disciplinary collaboration in informal learning environments. Lastly, the study will contribute to the theory and practice of developing learning communities for informal educators to engage in boundary crossing collaborations and professional development opportunities.

My Narratives and Positionality

For me, mathematics is a means of communicating with the world and understanding myself. It gives me joy, makes me frustrated at times, and fulfills my life. Being a female, I have faced challenges, doubts, and failures, but also joy, satisfaction, and success in my teaching and learning experiences with mathematics. In my experiences with learning advanced mathematics, teaching secondary mathematics, and conducting research in mathematics education, I consistently reflect on the gatekeeper role of mathematics, wondering who has a key to access the gate and who does not; I also wonder why this is the case. Though I have transformed my social status through pursuing mathematics, I am conscious that other people may be counted out of mathematics.

Being a Mathematics Learner

I was born in the 1980s. At that time, most people in China were poor, especially people who lived in the countryside like my family. I grew up in a very examination-oriented education system. As a girl born in a poor family in a rural area of China, I had always been told that being a high achiever in academia was the only way to change my life. My father was a community-sponsored middle school mathematics teacher whose salary depended on the year's harvest. Unfortunately, we usually had bad harvests. My mother had been deprived of educational opportunities during the cultural revolution; she did not even finish her elementary education.

I completed my six-year primary education in a rural village. The village did not have electricity until I was in fifth grade. I vividly remember working on math under the dim light of a kerosene lamp. My mother taught me the multiplication table (九九表), before I went to school. She taught me how to use an abacus to add, subtract, multiply, and divide. Basically, all the procedures relied on memorizing algorithms. My mother believes that these are not math, but life skills. She often says that if you know the formulas, you can go anywhere without being tricked (学会九九归, 走遍天下不吃亏). My father taught me math aligned with my school schedule. He asked me to solve math problems from exercises books. He scolded me a lot because I was not as quick as he expected. At times, he even said I was the stupidest kid he had ever met. I was greatly influenced by my mother, though she did not have many school experiences, which she has always regretted. She did not teach me *knowledge* directly, but she taught me to be persistent and optimistic, which greatly benefited me throughout my life.

I was a frustrated math learner in the early stage of elementary school. I remember I always made mistakes in computation, no matter how careful I was; I often misread numbers or operation signs. I was frightened about math class, homework, and tests. At school, I was teased by the teacher, who thought I should be good at math as a math teacher's daughter. My father was angry about my math performance at home because, as his daughter, I did not do well in math, which made him feel ashamed. I remember that he tried to tutor me but often ended up shouting at me. Tears and red Xs on my paper were my early memories of mathematics.

My mother was traumatized during the cultural revolution. She educated me to be a quiet girl. She told me to listen more, speak less, and never argue with people. If I had to say something, I needed to make sure it was absolutely correct. I was always hesitant to express my thoughts. I got used to listening to others first and then listening to my inner voice. Then, I interrogated my thinking by comparing it with others' thoughts. It was difficult for me to speak loudly, but I did not always accept the external voice. Instead, I struggled to accept or reject the ideas. I went back and forth to resolve the conflicts between my inner voice and external voice until I was completely convinced by my own voice or could not find any flaw in the external voice. It was a painful process; some issues perplexed me for years. However, mathematics has been a powerful part of my life because it was always straightforward, either right or wrong. I spoke up to defend my ideas in math and even argued with teachers, about which I never felt scared or ashamed. Mathematics allows me to raise my voice and offers me power.

In my early school years, I was very quiet and seldom raised my hand in class. This situation changed in my fifth grade when we started to learn applied mathematics. I did not know the word *logic* at the time, but I enjoyed this kind of math. I still made computational mistakes, but the teacher often called on me and asked me to explain my thinking to the whole class. I enjoyed explaining my thinking and answering questions from my teacher and peers, which I never did in other classes. I saw lights in their eyes when my explanation helped them make sense of the problems. That was my initial excitement about teaching in using a different way to explain problems, which can ignite a passion for learning.

From that time, I started to think about becoming a math teacher. At school, children were always dropping out of school to help their families. Unlike planting crops, education is a long process. The changes were so slow that they would go unnoticed from one year to the next. Maybe because people could not see benefits from education quickly, they gave up school too easily. At school, I kept losing friends. When I was in first grade, I had more than 20 classmates, but when I was in sixth grade, we only had four students, including me. Fortunately, my parents were insightful and forward-looking. They inspired and encouraged me to pursue higher education. I tried my best to help my classmates, see the beautiful and interesting parts of math. They liked to ask me math questions; I liked to help. Sometimes we discussed math with our teacher; sometimes, we did math tricks for fun. In my little heart, I tried to keep my friends in school, and I wanted them to have hope like me.

When I finished elementary education, I was selected to go to the county's best middle school, 40 miles away from home, based on my performance. The school had a few rural students like me, but many of my classmates had parents who were workers, teachers, or government officers. At school, communication was a problem for me. I found that I could share only a few things with my classmates. All I knew about was how to deal with a hard life. What they talked about was another world to me. I was curious about what others' lives looked like. Over time, I realized that not all people lived like us. I asked my parents why people who did not work as hard as us lived a better life than ours. They told me I could change my life through education. If I could pass a series of tests, I would get to college. Then I would find my value in society and achieve a better life. Looking back, high achievement helped me to access a different world. I know how difficult it was to get there. Only a few people who were from the same background as mine had opportunities for social mobility.

My school was the biggest middle school in the town, with 50 students in my classroom. Nobody, including my teachers, paid attention to a quiet girl. Until one day, my mathematics teacher noticed me because I used a very concise way to prove a geometry problem. He asked me to explain, and I explained very well. He said to the whole class, "*this girl has a math brain.*" From then on, my classmates called me *math brain*, and they came to me to ask for help. Mathematics offered me a way to find a position in the classroom. The teacher and other students identified me as a *math brain*, which greatly inspired me to work even harder in mathematics. I valued this identity, and I saw mathematics as my voice, so throughout my school life, maintaining high grades in mathematics was essential. Consequently, high achievement reinforced my mathematical identity throughout my year in secondary education. On the college entrance examination, my mathematics score was the highest in my school of more than 500 students.

Being a Mathematics Teacher

I did not struggle with most advanced mathematics courses in college, which made me think about becoming a mathematician. I applied for graduate school and spent three years pursuing a master's degree in pure mathematics, specifically in Representation Theory. However, I turned away from this direction because I felt isolated from both mathematics and people. When I worked with mathematics, I had to separate myself from what I was doing. I tried hard to eliminate emotions, feelings, and intuition and focused on being objective and neutral. I felt that I was dealing with something cold, challenging, and isolated. There was no connection between mathematics and me.

I spent most of my time on computation and proof, which was all individual work though we had a seminar in which I could present my ideas. However, in the seminar, I received far more critiques and questions than support. Therefore, I began to lose my confidence in mathematics. I was frustrated and started to doubt my decision to pursue mathematics. Looking back, I think many other students also had anxiety, but nobody spoke out about their feelings. The culture in the mathematics department made me feel ashamed to acknowledge something that I did not understand. The shame led me to recall my earlier experiences at school when I failed in computation. At that time, I could not see myself in mathematics. Although my advisor recognized my persistence and encouraged me to pursue a doctoral degree in Representation Theory, I decided to move into education. I chose to be a mathematics teacher as I imagined when I was a little girl. I loved both mathematics and teaching, so being a mathematics teacher was my ideal career choice. I was a successful mathematics teacher; students liked my teaching, their performance was excellent, and I was recognized with the Excellent Teaching Award many times in the school district.

In my ten years of secondary classroom math teaching, I gained a new understanding of mathematics: From my students, I learned that mathematics is a cultural practice. Years ago, I had a student in my class whose parents were vendors selling vegetables in a market near the school. After school, he had to help his parents in the market, and he did his homework there. Most of the time, he either could not finish or made a mess of his homework. He could not understand multiplication and division, which students were supposed to master in elementary school. What surprised me was that when I bought vegetables from him at the market, he did not use a calculator and never made mistakes. If he could do multiplication and division in selling, he should undoubtedly understand mathematics concepts and operations. Considering his study environment, I talked to his parents, and they allowed him to work on his homework in my office after school. I asked him why he could do mathematics in selling vegetables but could not understand school mathematics because, to me, they were the same thing. He told me they were completely different. Noticing my confusion, he explained to me. When he was doing school mathematics, he felt that he was dealing with numbers only related to grades. However, he did not care about grades because he was not good enough in everyone else's eyes. However, when he helped his parents sell vegetables, he was dealing with money that was directly related to his family earnings for that day.

I must say that he taught me a lesson. That was the first time I was aware of how far mathematics is from real life! After I heard the student's explanation, I adjusted my instructions to make them more relevant to his life. I told him that in multiplication, one number could be regarded as the price of a cucumber, and another number could be regarded as the quantity of the cucumbers. In division, the first number represented the money your customer paid, and the second number was the price of a cucumber. No surprise, he understood easily.

I also learned from talking with parents that mathematics is gendered. There was a girl in my class who was a top student and worked hard. There was a time when she was absent for several days. I contacted her parents, and they did not answer the call. When the girl returned to school, I was prepared to ask her why, but she was coming to say goodbye to me. She told me that her parents were running a small grocery store and that her mother was a deaf-mute who had difficulty communicating with the customers. Her father hardly managed the store by himself. The father told the girl that she had to drop out of school and help run the business. With the girl's guidance, I went to their grocery store and met her father there. I tried to persuade him to let the girl come back to school. The father told me that he had two children, and he could not let the other child drop out of school because he was a boy. There was no other choice for the father because he had to support the family. I was angry, but, at the same time, I understood the deep roots of gender stereotypes in our culture. I saw gender bias the father held as an individual case.

Later, I became aware of the existence of group gender stereotypes. I had led a *Paper Airplane Club* when I was a teacher in China. I introduced basic origami geometry in the club and gave students the freedom to design and test their paper airplanes. I opened the club to all students, but what surprised me was that only boys joined the club. Even though I strongly encouraged girls to join, they avoided it and told me it was a boys' club. That inspired my curiosity about how people can stereotype a pursuit to belong to either boys or girls. These experiences motivated me to investigate gender roles from a woman's perspective.

My Positionality as a Researcher

Mathematics empowers me to be a confident woman to communicate with the world while teaching mathematics provides me with a new perspective to understand mathematics. Coming to this study, I identify myself as a teacher educator, a researcher, a woman, and a mother. I enjoy these identities; being a researcher, which is challenging but fulfilling; being a mathematics teacher is the most comfortable identity in my life; being a woman and a mom rewards me with new perspectives about life and the world. As these identities bump up against each other and/or come together, I must confront many moral conflicts and feminist struggles. As a mathematics teacher, I struggle to speak up about inequities; being a woman, I have struggled because I had to leave my husband and study abroad for several years. Trying to be more independent and successful in my career, I need to maximize space for myself, especially time, which conflicts with being a mom. I try to find a way in my scholarship to resolve my feminist struggles and moral conflicts. I intend to be mindful of my role as a researcher, and I fully embrace my positionality as the subject and object of study.

For me, this dissertation study is a journey of self-redemption. On this journey, through exploring the relationships between women and mathematics, I carefully analyze voices, conflicts, resilience, and transformations of my participant and myself. Throughout this inquiry, I have had the opportunity to develop a collaborative personal and professional relationship with a female informal educator. In this experience, I have extended my empathy to a woman from Western culture; that is, a culture quite different from my own. During the study, I have learned and changed. My ingroup identity as a woman has been enhanced. I see myself as a normal individual woman belonging to multiple communities rather than categorizing myself as one of a few unconventional women.

Chapter Summary

In this chapter, I introduced the study, including a brief overview of the background related to women and mathematics/STEM education, which informed this research. I also outlined arguments concerning the need for and purposes of the study. It contains information about my narrative and positionality to illustrate how I come into this study, which also serves as a rationale for this work. Looking ahead, in Chapter 2, I provide a review of the relevant literature used to conceptualize the study and explicate the theoretical perspectives that shaped the study, and in Chapter 3, I describe the narrative inquiry methodology and affordances of the study. Chapter 4, “Looking Backward and Looking Forward: Becoming Female Educators,” presents Laura’s and my narratives of becoming women, teachers, and educators, which provides opportunities for us to reconstruct our past experiences and reflect on how narratives of histories shape our personal practical knowledge. In Chapter 5, “In the Midst of GEMS,” I explore GEMS as a space that empowers girls and women in STEM by telling stories of GEMS from leaders' perspectives. In Chapter 6, "Mathematics across Boundaries," I explore Laura's and my mathematics experiences and evolving views of mathematics to illuminate our conceptualizations of mathematics. In particular, utilizing a boundary crossing lens, I describe emerging mathematics in our collaboration and present the role of mathematics in GEMS. In the last chapter, "Discussion and Conclusion," I discuss the findings and revisit the theoretical framework and methodology. I also discuss the contributions and implications of the study and provide the conclusion for this dissertation.

CHAPTER 2. LITERATURE REVIEW AND THEORETICAL FRAMEWORKS

In this chapter, I review the relevant literature that informed this study. First, I describe research related to women in mathematics and STEM, especially studies that address women's career choices and identities in these fields. Next, I discuss the role of mathematics in the integration of STEM and spatial skills, followed by an introduction to research that addresses informal education, including informal learning environments and informal educators. Lastly, I describe my theoretical frameworks, which include a boundary crossing perspective and feminist theory. The boundary crossing perspective provides an analytic lens through which to understand the researcher and participant's boundary crossing experiences. Feminist theory provides a critical lens through which to investigate these experiences and advocate for girls and women's experiences in mathematics/STEM.

Women in Mathematics/STEM

Across the past five decades, researchers have conducted substantive investigations into gender differences in mathematics (e.g., Fennema & Sherman, 1977, 1978; Hyde et al., 1990; Lindberg et al., 2010; Moss-Racusin et al., 2021). Since the 1970s, overall gender differences in K-12 mathematics performance have become statistically insignificant in the United States (Lubienski & Ganley, 2017). Nevertheless, the perception that girls and women are less interested in mathematics and lack mathematical ability has persisted (e.g., Leder & Forgasz, 2010). This view has contributed to stereotypes that impact girls' and women's participation in mathematics/STEM (Dasgupta & Stout, 2014). In particular, girls and women at the intersection of multiple underrepresented communities are often stereotyped as less capable in mathematics/STEM (e.g., Joseph et al., 2019; Morton & Smith-Mutegi, 2018). Before I began this exploration, I did not realize that less women participating in mathematics fields is a problem. Exploring literature on women's career choices and mathematical identity opened my eyes on the issues of women in mathematics/STEM and women's position in society.

Career Choices

Utilizing various methods and sample populations, researchers have investigated factors that influence the career choices of women related to mathematics/STEM and have found that a person's conscious and unconscious choices reflect their personal values within a complex social reality. In other words, although women's career preferences seem like free choices, they are often constrained by social contexts (e.g., Brooks, 2007; Harding, 2012). From a social structural perspective, Eagly and Wood (1999) argued that "social change emerges, not from individuals' tendencies to maximize their inclusive fitness, but instead from their efforts to maximize their personal benefits and minimize their personal costs in their social and ecological settings" (p. 421).

Building on Eagly and Wood's (1999) social structure theory, Else-Quest et al. (2010) proposed the gender stratification hypothesis, which posits that cultural differences are related to girls' and women's opportunities in mathematics. Else-Quest et al. (2010) conducted a meta-analysis of two international assessments: the 2003 Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA), representing 493,495 students 14-16 years of age. They examined the magnitude of gender differences in mathematics achievement, attitudes, and affect across 69 nations, and found that when having female role models who excel in mathematics, girls would perform as well as their male classmates.

Women's career decisions to avoid mathematics may seem like a personal choice, but they are often constrained by messages from their surrounding environments that signal which directions are considered to be "naturally fit" for women and which directions would be a "dubious fit" (Dasgupta, 2011). Providing interventions, such as female role models, would affirm the possibility of women's success in mathematics/STEM and diminish the stereotype threats arising from other cues in an environment (e.g., Morton & Smith-Mutegi, 2018; Steele et al., 2002). Counternarratives of women in mathematics/STEM fields can be used to mitigate the effects of negative stereotypes (Dasgupta & Asgari, 2004; Diekman et al., 2010).

Some scholars have contended that women tend not to choose STEM fields because they prefer working with people (i.e., people-oriented) rather than working with things (e.g., Diekman et al., 2010; Looker & Magee, 2000, Su et al., 2009). The values attached to STEM characteristics in society contribute to the perceived available options for an individual's career selection (Eccles, 1994). High school girls' preference for work that is people-oriented predicted entrance into people-oriented STEM fields (i.e., health, biological, and medical sciences) instead of other STEM fields (Eccles & Wang, 2016). Diekman et al. (2010) speculated that incompatibility with a preference for working with people might deter women from some STEM careers. Society expects women to place more value on taking care of family, helping others, and developing interpersonal relationships. In contrast, men are often expected to prioritize completing challenging tasks, promoting social status, and using mathematics and computers in work (Dasgupta & Stout, 2014; Eccles, 2009). These trends could push people in the opposite direction, that is, gender role-related experiences could lead gendered individuals to develop different personal values (Dasgupta & Stout, 2014).

Mathematical Identity

Eccles (2009) defined personal identities as values that "serve the psychological function of making one feel unique" (p. 78). In contrast, collective identities are a "personally valued part of the self that serves to strengthen one's ties to a highly valued social group and relationships—such as one's gender, race, religion, social class, culture, and family" (p. 78). Specific practices can construct personal and collective identities. As such, when the practices convey subjective value, individuals will often be motivated to participate in these practices and reinforce their salient identities. Gender as a social construct is a collective identity. Thus, specific activities, behaviors, and practices are attached to gender roles, and gender behavior norms reinforce gender identity and potentially give rise to gender stereotypes (Steele et al., 2002).

Mathematical identity can be conceptualized and defined in multiple ways. Drawing on Lave and Wenger's (1991) situated learning perspective, mathematical identity refers to participation in mathematical communities of practice and is linked to learning and becoming a certain kind of person by doing activities perceived as mathematical in social practice. Other researchers have defined mathematical identity as beliefs about mathematical ability, knowledge, and learning strategy, mostly referring to school mathematics (e.g., Martin, 2000). Mathematical identity can also be viewed through a narrative lens, defined as stories people tell about themselves or tell through other related people (e.g., Sfard & Prusak, 2005). In addition, researchers emphasize that an individual's mathematical identity explicitly connects with their other social identities, such as culture, gender, language, and race (e.g., Gutiérrez, 2012).

Parents, teachers, and peers communicate stereotypes to girls that have the potential to impact their mathematical identity. Parents of boys often provide activities and stimuli such as telescopes, microscopes, scientific kits, and reference materials (Hill et al., 2010); whereas, parents might encourage daughters to provide caring service to other people (Diekman et al., 2010). Moreover, stereotypes of girls and women having inferior mathematics abilities may shape teachers' evaluations of their students (Riegle-Crumb & Humphries, 2012). Teachers often use *hard-working* to refer to girls but use *bright* to refer to boys who perform at the same level, which has the potential to constrain girls' mathematical identity (Walkerdine, 1998). A recent study revealed that teachers' gendered views on mathematics performance might prevent girls from developing a positive identity in mathematics, thus excluding girls from becoming high-level mathematics students (Jaremus et al., 2020). Additionally, the attitudes and behaviors of male peers, such as taking over girls' work or belittling their contributions, impede girls' development of positive mathematical identity (Allan & Madden, 2006).

Researchers have suggested that mathematical identity is often closely connected to mathematics learning experiences (e.g., Boaler & Greeno, 2000; Langer-Osuna & Esmonde, 2017; Sam & Ernest, 2000) and interacts with other identities, such as sexual identity (e.g., Wiest, 2021) and racial and gender identity (e.g., Morton & Smith-Mutegi, 2018). Students participate in mathematics through interactions with teachers and peers, which later contributes to how they see themselves in the context of mathematics and how they describe themselves mathematically to other people (Sam & Ernest, 2000). In addition to learning mathematics, students also construct multiple identities in mathematics. Thus, without addressing sociopolitical concerns, mathematics education continues to advantage privileged groups (Langer-Osuna & Esmonde, 2017).

Nature of Mathematics

What mathematics is and how to teach it are questions that mathematics educators constantly confront. Educators' viewpoints of the nature of mathematics determine whether they can recognize and exploit learning opportunities for students (Burton, 2002). I began rethinking these questions as I started to work with Laura. At first, I doubted that GEMS work was related to mathematics or mathematics education. As I continued seeking the answers in GEMS, my view of mathematics and mathematics teaching began evolving.

Public Image of Mathematics

Mathematics is often perceived as “difficult, cold, abstract, and in many cultures, largely masculine” (Ernest, 1996, p. 802). Sam and Ernest (2000) conceptualized the image of mathematics as “a mental representation or view of mathematics, presumably constructed as a result of social experiences, mediated through school, parents, peers or mass media” (p. 195). They surveyed 548 British adults including people directly and indirectly involved in mathematics education and interviewed a sub-sample of 62 adults. The findings revealed that most people’s image of mathematics is close to their image of learning mathematics, which suggests a connection between people’s images of mathematics and their school experiences with mathematics. Upon further analysis of the interview data, they found that a person’s school learning experience was one of the major factors for the formation of their image of mathematics. Particularly, mathematics teachers greatly influence people’s image of mathematics. Sam and Ernest (2000) suspected that the widespread negative public image of mathematics might be a result of problems in mathematics education.

The images of mathematics and mathematicians in popular culture and literature can reflect public opinion of mathematics and mathematicians. Academic literature concerning women and mathematics usually depicts female mathematicians and scientists as women with little concern about the nature of their work and scientific contributions (Damarin, 2008). Darragh (2018) examined young adult fiction and found that school mathematics was described as obligatory and portrayed as tense, terrible, difficult, and less useful/not useful; likewise, mathematics classes were portrayed as tense, terrible, difficult, and different.

Mendick (2005) analyzed portrayals of mathematicians, both males and females, in popular movies and found that they were exceptional people described as genius and born in a way that distinguished them from ordinary people. Later, Epstein et al. (2010) made a critical analysis of popular cultural texts drawing from a variety of sources, such as movies, television, novels about mathematics and mathematicians and examined the images of mathematics and mathematicians hold by young people including 556 Year 10 graders and 100 sophomore college students in the UK. They found a close resemblance between young people’s images of mathematics and mathematicians and images widely dispersed in popular culture.

Though people’s images of mathematics are greatly influenced by their school mathematics learning experiences, our recent study with the original group of GEMS girls (OGGs) indicated that their out-of-school learning experiences with alternative curricula and approaches had the potential to disrupt the traditional image of mathematics (Zhou et al., 2021). By intentionally implementing relevant and engaging tasks which do not necessarily focus on results, mathematics teachers could change the image of mathematics to one that is relevant to students. In addition, fostering collaborations between schools and other organization (e.g., universities, STEM programs) could potentially broaden individual and public images of mathematics (Zhou et al., 2021). Exploring mathematics across contexts could provide a way to disseminate the public's alternative images of mathematics (Nemirovsky et al., 2017).

Humanizing Mathematics

Given the growth of careers in STEM and the significant role of mathematics in STEM, reframing and humanizing mathematics as normal, ordinary, and relevant to people is an urgent task for the mathematics education community (e.g., Darragh, 2018; Sealey & Noyes, 2010, Stephan et al., 2015). Specifically, Stephan and colleagues (2015) reported "grand challenges" for mathematics education, including "changing the public's perception about the role of mathematics in society," "achieving equity in mathematics education," and "changing perceptions about what it means to do mathematics" (p. 139).

Researchers have used the concept of *humanizing mathematics* to highlight the human practice characteristic of mathematics, shifting the public image of doing mathematics as special people doing extraordinary work (Mendick, 2005). Though a substantial amount of literature uses the term *humanizing mathematics*, there are still no agreed-upon descriptions of what humanized mathematics is and how to humanize mathematics. As early as the 1970s, researchers started to seek ways to humanize mathematics. Wheeler (1975) described humanizing mathematics as fostering students' awareness of self and the world through insights into mathematics, which he explicated as "the act of attention that preserves the significant parts of the experience, that pegs and holds them in the self so that they are available for future use" (p. 8).

Falkenberg (2006) conceptualized humanized mathematics as a human activity that is something humans do within particular social contexts. From an equity in mathematics perspective, Gutiérrez (2012) used a window/mirror metaphor to describe the impact of the mathematics curriculum on one's identity. Students can see themselves in the mirror (mathematics) as well as have a broader view of the world through the window (mathematics). What is more, she claimed that "using mathematics to analyze social justice issues might offer a mirror to students who have been marginalized by society, while it provides a window to students who benefit from the status quo" (p. 20). Gutiérrez (2018) brought up the term *rehumanizing mathematics* to advocate for developing practices for those historically marginalized communities (i.e., Black, Indigenous and Latinx). She claimed that "a student should be able to feel whole as a person—to draw upon all of their cultural and linguistic resources—while participating in school mathematics" (p. 1).

Other scholars humanized mathematics by incorporating aspects of students' cultural and community into mathematics teaching and curricula (D'Ambrosio, 1985; Rosa & Orey, 2008). D'Ambrosio (1985) introduced the term *ethnomathematics*, which refers to mathematical practices in identifiable cultural groups and recognizes the development of mathematics as embedded in a cultural context. That is, people within various cultural groups develop unique methods and techniques to understand their realities in response to the problems they encounter (Rosa & Orey, 2008). D'Ambrosio (2015) suggested that teachers need to help students recognize their mathematical potential by acknowledging students' cultural value and bringing culturally relevant mathematics into teaching. In 2001, he cautioned that without a culturally relevant curriculum, most students leave school thinking that mathematics is something to be done only at school and has no relevance to their lives. Nevertheless, school mathematics often focuses on the mastery of skills and accumulation of facts and rules to prepare for standardized tests (Cooper, 2011). Conversely, mathematics in other contexts (i.e., designed informal learning environments) is not typically accompanied by traditional academic assessment, allowing students to explore relevant mathematics and experience satisfaction in mathematics learning (Zhou et al., 2021; Nemirovsky et al., 2017).

The word *humanizing* indicates a changing and ongoing process that requires a responsive curriculum for learners and evolving contexts. In this study, my view of mathematics changed from mathematics as an individual school discipline to mathematics as socially constructed knowledge and the teaching of mathematics as a human activity that is embedded in social contexts. I conceptualize *humanizing mathematics* as a process that blurs the boundary between mathematics in school and other practices, which allows teachers and students to see mathematics learning as ordinary human activity rather than specialized work completed in school.

Mathematics and STEM

The role of mathematics in STEM education is significant; students who do not take advanced-level mathematics courses in high school will likely not select mathematics-related majors in college (Watt et al., 2017). Students' mathematical perception and mathematical identity can predict their career path, that is, choosing mathematics-related majors or not (e.g., Correll, 2001; Eccles, 1994, 2006). Sax et al. (2015) pointed out that women's underrepresentation in STEM fields is often a result of their consistently lower mathematics self-concepts.

The Role of Mathematics in STEM

Although mathematics provides foundational knowledge for other related disciplines and is used as a common language underpinning engineering, science, and technology, it is often positioned in integrated STEM in a supporting role and has not received significant attention, even having been abandoned after solving the problem (Civil, 2007). Thus, without purposeful design, mathematics is often utilized in an incidental or instrumental way rather than used to develop an understanding of mathematical concepts in STEM integration (Fitzallen, 2015). Shaughnessy (2013) called for mathematics educators to highlight significant mathematics in STEM integration; otherwise, mathematics will remain silent in STEM initiatives and approaches. He suggested making "mathematics transparent and explicit" (p. 324) throughout its integration with other STEM disciplines.

Researchers have been exploring ways to promote the role of mathematics in integrated STEM. For example, Shaughnessy (2013) proposed three aspects of integrated STEM activities that can foreground mathematics: "(1) there must be a problem to solve, (2) there must be significant mathematics involved in the problem, and (3) the problem should require teamwork that draws on knowledge and approaches from several disciplines" (p. 324). Others have suggested designing integrated STEM activities that explicitly address mathematics concepts through integration with at least one of the other disciplines of STEM or explicit mathematics in STEM contexts (e.g., Stohlmann, 2018, 2019). Mathematical modeling has also been recognized as a way to connect to the real world by addressing real-world problems, which aligns with STEM education's aim of using application knowledge to solve problems (e.g., Common Core State Standards Initiative, 2010; Maass et al., 2019; Stohlmann, 2019).

Focusing on mathematics teaching, researchers suggested developing mathematics teachers' 21st-century competencies to advance mathematics in integrated STEM education (e.g., Maass et al., 2019; Rahman et al., 2021). English (2017) stated that mathematics is often taught in isolation from the real world, which makes highlighting mathematics in STEM difficult for teachers. Beswick and Fraser (2019) identified teachers' 21st-century competencies for teaching in STEM contexts, including core content knowledge, cross-disciplinary knowledge, problem-solving and critical thinking, communication and collaboration, creativity and innovation, etc. Among these competencies, cross-disciplinary knowledge requires synthesizing knowledge across multiple disciplines and using various forms of representations to convey integrated knowledge. Beswick and Fraser also remarked that current mathematics teaching does not inspire students who are potential STEM graduates to pursue STEM careers that are beneficial for individuals and society. Using integrated approaches to teaching mathematics requires teachers to have further knowledge, which is beyond pedagogical content knowledge for teaching single subjects (Shulman, 1986) and specific content knowledge in mathematics (Ball et al., 2008).

An emphasis on integrated STEM influences mathematics education in multiple ways. STEM integration can promote students' mathematics performance because it focuses on meaningful integration within authentic contexts and real-world problems (Stohlmann, 2018). In addition, STEM education provides cross-curricular opportunities to support students' mathematical development. Miller (2019) reported that elementary students developed higher-level mathematical thinking through participation in a short-term coding program. Using an integrated STEM approach to learning mathematics is an alternative approach for encouraging students' interest in mathematics (English, 2016, 2017). Nevertheless, the reciprocal relationship between mathematics and integrated STEM has not been recognized fully. Compared with well-established single subjects, STEM integration is still in the embryonic stage. More research is needed to explore the relationships between teaching and learning mathematics and STEM integration, and the connections between mathematics and other subjects (i.e., science, technology, and engineering).

Spatial Thinking

Gender differences in complex problem-solving and spatial skills have been consistently identified, with boys and men outperforming girls and women (Corbett & Hill, 2015; Hyde et al., 1990; Lindberg et al., 2010; Sorby, 2001). Spatial thinking is essential for success in mathematics and science and has a positive relation with STEM learning at an early age (e.g., Gunderson et al., 2012; Uttal & Cohen, 2012; Uttal et al., 2013). Gunderson et al. (2012) stated that children's spatial skills (i.e., mental transformation ability) during the early grades can predict their understanding of numerical knowledge. This finding echoes Geary and Burlingham-Dubree's (1989) claim that preschoolers' spatial skills are positively correlated with their strategy use in an arithmetic task.

Previous studies have confirmed that spatial skills are not inherited and fixed but can be improved with appropriate spatial training (e.g., Sorby, 2001, 2018; Uttal & Cohen, 2012; Verdine et al., 2014). For early-age children, spatial tasks are often demonstrated as play activities, such as block-building activities, rather than formal tasks. Uttal et al (2013) investigated the malleability of spatial thinking and sought ways to improve spatial thinking. They found that spatial training—including video games, spatial tasks, and course training—can improve students' spatial skills. More importantly, spatial training was durable, meaning that the effect of training persisted for several months. The improved skills gained from spatial training can be used to solve similar tasks in other contexts (Uttal et al., 2013).

Uttal and Cohen (2012) asserted that spatial training especially is important to promote spatial skills for people who are starting to get into STEM fields. Other researchers also supported that spatial skills are an important predictor of STEM achievement, choosing STEM majors, and pursuing STEM careers (e.g., Sorby et al., 2018; Stieff & Uttal, 2015; Wai et al., 2009); students who are less confident in their spatial skills are unlikely to choose an engineering major later (Sorby, 2001, 2018). Applying spatial skills becomes more critical in advanced mathematics courses such as geometry and calculus, which are required for most STEM domains (Ganley & Vasilyeva, 2011).

Mathematics standards and principles encourage the development of students' spatial skills through hands-on experiences with physical tools and materials and the use of technology such as dynamic software (e.g., CCSSI, 2010; NCTM, 2000, Zhou et al., 2022). However, there is a lack of existing spatial curricula that include activities and instruction for teachers to use for everyday teaching (Power & Sorby, 2021). Developing spatial curriculum also often fails to incorporate the perspectives of practitioners, which limits the effectiveness of the curriculum (Power & Sorby, 2021). Due to the critical role of spatial skills in STEM, gender imbalance in spatial skills plays a crucial part in reproducing gender inequality in STEM fields (Else-Quest et al., 2010). Thus, there is a need to incorporate and elevate female practitioners' voices in developing and implementing spatial curricula (Power & Sorby, 2021).

In this dissertation, I aim to address the issue of women's lower participation in mathematics-intensive STEM fields. In the exploration, I cross the boundary between mathematics and STEM and gain a broad understanding of mathematics that connects with other STEM disciplines. Unpacking the inherent relations between mathematics and STEM contributes to a better understanding of gender inequity in mathematics/STEM and recognition of mathematics in informal STEM learning environments. Particularly, exploring literature on spatial skills curricula helped me to better support Laura in developing and implementing a spatial curriculum in GEMS.

Informal Education

Informal learning environment is often used as a general reference to a learning setting that is different from school. Still, there are no agreed-upon definitions in the literature. The National Research Council [NRC] (2010) described three categories of informal science learning environments: (a) everyday informal environments such as family discussion, pursuing a hobby, and daily conversation; (b) designed environments such as museums, science centers, aquariums, and libraries; and (c) programming, such as after-school programs, clubs, and summer programs.

Substantial work has been done to pinpoint the effectiveness of well-designed programs in increasing girls' interest and shaping their identities in STEM areas (e.g., Chen et al., 2011, Chittum et al., 2017). These programs often provide a variety of successful interventions, such as female role models, hands-on activities, and single-sex learning environments (e.g., Holmes et al., 2012). In particular, girls are most eager and participatory in teams that have either gender parity or a female majority and less engaged in teams with female minorities (Dasgupta & Stout, 2014).

Informal Learning Environments

Unlike school settings, which tend to prepare students for standardized tests and focus on improving individuals' academic performance, informal mathematics learning is designed to allow students to experience socialization into their collective identities (e.g., gender, race, intersection of race and gender) (e.g., Copper, 2011; Smith-Mutegi & Morton, 2021). Informal mathematics education takes place in a variety of socio-cultural contexts (Civil, 2007). People use flexible strategies in diverse settings to solve mathematical problems outside of school, such as vendors using mathematics in the marketplace, which are often significantly different from those taught in school (e.g., Lave, 1988, 2019; Nunes et al., 1993; Saxe, 1988). Nunes (1999) pointed out that concepts, methods, and procedures are often the goals for instruction in school mathematics—solving problems for the teacher's sake. In informal learning, by contrast, mathematical methods remain a means of solving problems for the sake of students' personal interests; such solutions view mathematics as practical rather than theoretical, which allows for greater flexibility (Lave, 2019).

However, discrepancies exist between informal education and school education. Though informal mathematics involves the development of mathematics concepts, people perceive it differently from formal mathematics because it is often embedded in authentic contexts. In Nunes et al.'s study (1993), children who successfully solved problems in authentic contexts failed in school mathematics that involved similar mathematical thinking and concepts. The traditional view of mathematics—which values abstract over concrete, formal over informal, and theory over practice—is socially ingrained (Ernest, 1991). As such, "informal mathematics education is an emerging field of learning with the unique potential to disseminate alternative images about the nature of mathematics and to realize the potential for everyone to engage with mathematics in creative and diverse ways" (Nemirovsky et al., 2017, p. 975). Therefore, informal learning environments have the potential to consider social, cultural, and historical issues and barriers for underrepresented students, thus opening a new way of thinking about equity in mathematics education.

NRC (2015) described several criteria of productive informal STEM programs that "(a) engage young people intellectually, socially, and emotionally; (b) respond to young people's interests, experiences, and cultural practices; and (c) connect STEM learning in out-of-school, school, home, and other settings" (p. 15). Nemirovsky et al. (2017) provided a more specific definition of informal mathematics learning environments as "intentionally designed to support mathematics learning, whether because they are structured through programs with regular schedules and assigned educators or because they host technologies, tools, or exhibits designed to engage the user with mathematics" (p. 970).

In this study, drawing from reviewed literature, my understanding of an informal learning environment included a planned curriculum and intended pedagogies to support learners in developing a positive mathematical/STEM identity, more specifically referring to designed learning environments (i.e., after-school programs as mentioned by NRC).

Informal Educator

In general, informal educators are educators in informal settings. But, like informal learning, there is no consistent description of informal educators and research on informal educators is particularly limited. Thus, informal educators' learning experiences, teaching, reflective practice, and professional support are rarely discussed in the literature (Allen & Crowley, 2014, Jeffs & Smith, 2021). In this study, I seek to contribute to the emerging conversation about informal educators. I use *informal educator* not to indicate a dichotomy between informal and formal. Rather, informal educators are educators who engage in informal settings and bring experiences and motivation in the social exchange with learners.

The educational landscape has shifted from relying on the school to provide various educational opportunities outside of school contexts, to teaching the individual subjects, to emphasizing the integration of multiple subjects, which has led to the increasingly important role of educators to cross disciplines and context boundaries. Integration between science, technology, engineering, and mathematics creates a different kind of content knowledge from single subjects. Thus, informal STEM educators, who teach content for which they do not have sufficient preparation and specialization, encounter "teaching out-of-field" (Luft et al., 2020, p. 719). Although most informal educators have a college degree, they often find it challenging to facilitate and teach science concepts, particularly with regard to content and pedagogy (Ennes et al., 2020).

Nevertheless, informal educators often have few opportunities to receive professional support, such as mentoring and training (Allen & Crowley, 2014; Jeffs & Smith, 2021). In addition, they are often isolated in their practices, with few opportunities to discuss the details of their work with colleagues who have expertise in leading informal learning (Allen & Crowley, 2014). Without training and reflective practice, informal educators are likely to continue to lead informal learning in a manner that mirrors their prior experiences in informal environments or their own teaching in the classroom (Allen & Crowley, 2014).

Therefore, like formal teachers, informal educators need professional communities to develop professional vocabulary and culturally responsive pedagogy that help them lead informal learning, particularly for learners who are underserved in formal educational settings (Bell et al., 2009; Castle, 2006; Ennes et al., 2020). The development of informal educators' approaches is different from formal teacher preparation programs. Beyond content and subject knowledge, informal educators have more requirements to adapt to the flexible and emerging opportunities in informal learning environments (Nemirovsky, 2017).

Jeffs and Smith (2021) sought to portray the educational imagination of informal educators. Building on common characteristics of educators (formal and informal), they noted that informal educators do not have to be polymaths. Rather, they should have rich and broad knowledge of general education and be passionate about new ideas, information, and resources. Specifically, Nemirovsky (2017) described informal mathematics educators as those who

> engage learners in the development of skills with a wide range of tools and materials; who facilitate the learners' involvement in the design of their own collective projects; who [are] eager to pursue ideas [they] had not expected to have arisen; who habitually trespass conventional boundaries between mathematics, art, history, design, literature, sports, music, or philosophy. (p. 977)

In this study, Laura, self-identified as an informal educator, crosses boundaries between multiple disciplines and leads and facilitates mathematics/STEM learning in GEMS as an informal learning environment. In the language of Lave and Wenger (1991), I have grown from an outsider to a peripheral participant in the informal learning community, and eventually I will feel comfortable calling myself an informal educator. Through exploring Laura's experience as an informal educator and interrogating my own development as an informal educator, I intend to highlight the experiences of informal educators.

Boundary Crossing Perspective

The boundary crossing perspective and Feminist theory informed my theoretical frameworks in this narrative inquiry study. The boundary crossing perspective offers an analytical lens to understand people's experiences in crossing different communities and activity systems, and Feminist theory provides a critical lens to explore power relations behind women's experiences regarding discontinuities and continuities in crossing various boundaries.

Boundary crossing has become an explicit concept in communities of practice (Wenger, 1998) and the theory of expansive learning (Engestrom, 1987/2015). Though these two theories are rooted in different learning theories (i.e., situated learning theory and sociocultural learning theory, respectively), their descriptions of boundary crossing are compatible and coincide with my views of learning. Specialization in education fields results in an "increasing need to cross boundaries because today's complex problems frequently require solutions that are not confined to any one practice, or even to a single organization" (Wenger et al., 2002, p. 153). As learners are engaged in learning as whole persons, crossing boundaries between practices and communities becomes more frequent, enhancing learning processes. Akkerman and Bakker (2011) contended that the nature of boundaries is ambiguity, "both–and as well as neither–nor phenomena at the same time" (p. 150). Meanwhile, Akkerman and Bakker proposed that learners may experience a sociocultural difference or disconnection that leads to discontinuity in action or interaction. To illustrate learning as boundary crossing, the following section discusses two core components of the boundary crossing perspective: horizontal learning and learning at boundaries.

Horizontal Learning

Sfard (1998) noted two metaphors for learning, the acquisition metaphor and the participation metaphor. The acquisition metaphor highlights how a student acquires knowledge about the world, and the participation metaphor focuses on the enculturation of the student in a particular community. Rather than competing, these metaphors offer different perspectives which are presented simultaneously in most learning contexts. The two metaphors have foundations in learning theories. For instance, from the cognitive development perspective, learning refers to cognitive processes and ability changes. From a situated learning perspective, the learning process includes all components of a person's body, mind, emotions, and social relations as they participate in communities of practice. Learning then alters a learner's identities (Lave & Wenger, 1991; Wenger, 1998).

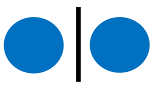
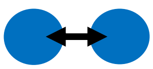
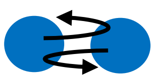
Engestrom (2001) was unsatisfied with the two metaphors of learning and pointed out that they both describe learning as a vertical process in a stable given system. He argued that learning is a horizontal movement, in which a learner should be approached as a whole person who participates in school and many other practices. One can simultaneously participate in multiple communities of practices or multiple social systems and move from one system to another. For example, one's classroom experiences are not isolated from experiences in out-of-school contexts and everyday life. Learning across different contexts leads to continuities as well as discontinuities (Akkerman & Bakker, 2011). Continuities refer to individuals' experiences in one context that relate to their experiences in another context while discontinuities suggest that individuals' experiences disconnect or even conflict with their various contexts of participation (Bronkhorst & Akkerman, 2006). Students' academic and non-academic interests are found not to be fixed and independent in their development. Some interests are shared across different contexts, and others differ in across-content continuity (Akkerman & Bakker, 2019). The horizontal aspect of learning provides a powerful lens for analyzing interactions between different activity systems which Akkerman and Bakker (2011) conceptualized as boundary crossing.

Learning at Boundaries

All learning involves boundary crossing. Wenger et al. (2002) argued that the "learning potential of an organization lies in this balancing act between well-developed communities and active boundary management" (Wenger p. 154). Akkerman and Bakker (2011) called this "dialogical learning" and identified four learning mechanisms at the boundaries (i.e., identification, coordination, reflection, and transformation). In the identification process, boundaries are encountered, and people in the boundaries re-identify themselves without necessarily overcoming discontinuities. In the coordination process, learning aims to overcome the boundaries, and continuity is deliberately established, facilitating movement between different communities. Reflection furthers identification and focuses on reconstructing identities to inform future practice. Akkerman and Bakker (2011) built on Boland and Tenkasi's (1995) concepts of perspective making and perspective taking to describe reflection. Perspective making means one's understanding, and knowledge of a particular issue is explicit. Perspective-taking means "taking of the other into account, in light of a reflexive knowledge of one's own perspective" (Boland & Tenkasi, 1995, p. 362). Transformation can entail the emergence of new in-between practices, which are difficult to achieve and imply sustainable impact.

Akkerman and Bruining (2016) further conceptualized the four learning mechanisms and suggested that boundary crossing in each form can take place at institutional, interpersonal, and intrapersonal levels. For this study, I only focus on boundary crossing at the interpersonal and intrapersonal levels (i.e., interpersonal experiences between researcher and participant and intrapersonal experiences of individual researcher or participant) (see Table 1). At an interpersonal level, boundary crossing is about actions and interactions of people from multiple groups engaged in the groups' associated practices. In this study, I come from a researcher and a teacher educator position, and Laura comes from a practitioner position. We established collaborative relationships to work together on GEMS. At an intrapersonal level, an individual participates in more than one separate practice. For example, in conducting this study, I participate in educator, researcher, and leadership roles.

Table 1. Boundary Crossing Framework

Learning Mechanism	At the Interpersonal Level (Action and Interaction Between Actors from Different Practices)	At the Intrapersonal Level (Participation of a Person in Two or More Practices)
Identification 	People come to (re)define their different and complementary roles and tasks.	A person comes to define [*their*] own simultaneous but distinctive participatory positions.
Coordination 	People seek shared means or procedures for exchange and cooperative work.	A person seeks means or procedures to distribute or align [*their*] own participatory positions in multiple practices.
Reflection 	People come to value and take up another's perspective.	A person comes to look differently at their own participatory position because of the other participatory position.
Transformation 	People face a shared problem, start collaborative work, and may build group identity.	A person develops a hybridized position in which previously distinctive ways of thinking, doing, communicating, and feeling are integrated.

Extracted from (Akkerman & Bruining, 2016, p. 246)

Crossing Boundaries in-and out-of-School

Students are often assumed to lack relevant knowledge and skills in a schooling system rather than being seen as people who have interconnected identities and as students who are simultaneously engaged in multiple practices and communities (Akkerman & Eijeck, 2013). Understanding the existence of boundaries is not enough to understand students' learning; whether and how boundaries relate to discontinuities in the learning process needs attention. Bronkhorst and Akkerman (2016) synthesized 186 empirical studies to investigate continuity and discontinuity in students' learning across in-school and out-of-school boundaries. They found that some out-of-school designed practices were intended to establish continuity. For instance, some informal educational settings provide authentic and flexible contexts, offering more engaging learning. At the same time, intended discontinuity was found to maintain distance and difference between school and out-of-school because students disengage from some school practices and wish to not align with these practices. Discontinuity was often found for underrepresented students, resulting in disengagement with school. For instance, students' academic and non-academic interests revealed discontinuity across contexts. The development of interests is not fixed or in a linear pattern across school and out-of-school contexts (Akkerman & Bakker, 2019). Intended discontinuity challenges the widespread preference for continuity between school and out-of-school.

In addition to the deliberate continuity, continuity can occur as a given in and out of school. Even in everyday life, students perceive what counts as learning and what forms of communication count as valuable and effective in education, influenced by the notion of school learning (Akkerman & Eijeck, 2013). In other words, boundaries between school and out-of-school maintained at the schooling system can easily be enacted at the individual level of students and inform students' identities. Therefore, conscious recognition of boundaries informs the creation of a learning environment that appreciates students as whole people. Meanwhile, recognition also engages teachers in the social practice of inquiry into learning to identify multiple identities of students (Akkerman & Eijeck, 2013).

In this study, GEMS is an out-of-school learning space including only girls, which establishes an intended discontinuity with the school setting. Regarding organization and learning, GEMS maintains different practices from school. Thus, boundary between learning in GEMS and school learning is expected.

Crossing Boundaries in Mathematics/STEM Education

Though mathematical practices are frequently used in the context of STEM, mathematics is not often the center of attention and is sometimes even hidden in the STEM context (Civil, 2007; Shaughnessy, 2013). Boundary crossings between disciplines include interventions, the use of tools, and the notion of changes in learning behavior (Dillion, 2008). Researchers have been searching for ways to promote boundary crossing between mathematics and integrated STEM (e.g., English, 2017; Fitzallen, 2015; Stohlmann, 2019). Mathematical modeling is widely recognized as an effective approach to boundary crossing between mathematics and STEM (Carreira & Baioa, 2018; Juardak, 2016). Specifically, inquiry-based mathematical pedagogy and uncertainty of the boundary crossing between mathematics and STEM inform STEM practices (Leung, 2019). Uncertainties would inspire students to seek different mathematical strategies and develop their divergent thinking. In addition, collaboration among STEM teachers could expand their pedagogical content knowledge, which is a critical step for boundary crossing in STEM education (Leung, 2020). Informal STEM educators usually engage in numerous communities; identifying their boundary crossing experiences and pertinent professional development activities leads them to develop high-quality teaching practices (Swanson, 2018). In this study, my participant and I have significant boundary crossing experiences; unpacking these experiences will lead to a better understanding of mathematics in informal STEM education.

Some mathematics education researchers have framed their studies using boundary crossing in teacher education programs (e.g., Butler et al., 2019; Goos, 2015, 2018). For example, Goos (2015) investigated boundary crossing between mathematicians and mathematics educators in pre-service teacher education, focusing on conditions that enable or hinder boundary crossing and learning mechanisms at the boundaries. She found open-mindedness, mutual respect, and shared values were crucial to enabling interdisciplinary collaboration while cultural differences were hindrances to broader collaboration. Transformation learning mechanisms emerged at the boundaries with the aspects of sharing problem space, maintaining the uniqueness of their established practices, and continuous joint actions at the boundary (Goos, 2018). The literature I reviewed above using boundary crossing perspective to investigate learning and collaboration in boundaries provided examples for this study and informed me about viewing my experiences through a boundary crossing lens.

Feminist Theory

Feminist theorists challenge the dominant form of knowledge and seek to eliminate boundaries built by people who occupy privileged positions (Hesse-Biber, 2012). Through claiming knowledge, feminist researchers continue to inquire who can be a knower and what can be known (Harding, 1991). Starting from women's lives and experiences as a source of knowledge, researchers drawing on feminist theory aim to make sense of women's situations and point to effective strategies for social change. Thus, feminist theory focuses on creating a group consciousness that enables feminist struggle and further highlights and fosters feminist perceptions (Harding, 2012).

Feminist theorists do not assume that all women confront common oppression or struggles. Rather, they acknowledge differences in women's experiences related to, among other things, gender, race, class, sexuality, and culture, and strive for methodological commonalities

(Hirschmann, 1998). Feminist researchers bring women's voices from the margin to the center and develop "more nuanced understandings of the multiple power relations that shape women's situations" (McCann & Kim, 2013, p. 25). The use of feminist theory enables researchers to bring into view how social institutions affect everyday lives and uncover possibilities for change (Hesse-Biber & Piatelli, 2012). The research questions that feminist researchers ask are designed to position women's voices and lives of other marginalized groups at the center of social inquiry (Hesse-Biber, 2012).

Conceptualizing Feminist Theory

A wave metaphor has been used within U.S. feminist theorizing to represent historical women's movements. The conventional periodization of the first wave marks the passage of women's suffrage in 1920 (Jagger, 2016). The second wave of feminism emerged in late 1960 rooted in equal rights and women's liberation (Baxandall & Gordon, 2002). For instance, during this second wave, Title IX was passed and expanded to eliminate gender-based discrimination in education which greatly influenced women's participation in sports and educational fields (Rolison, 2003, Walters & McNeely, 2010). Some scholars have suggested that a wave metaphor is not suitable to describe today's feminist activism (e.g., Evans, 1995, Jagger, 2016, Nicholson, 2010). However, the term second wave of feminism is still useful to describe the historical era. Nicholson (2010) stated:

> it was very useful for feminists to begin to describe their movement as the "second wave" of feminism. It was useful because it reminded people that the then current women's rights and women's liberation movements had a venerable past—that these movements were not historical aberrations but were part of a long tradition of activism. (p. 34)

Intersectional feminism (Collins, 2019; Crenshaw, 1993) emerged to challenge "common women's experience" and seeks to understand gender in relation to other identities such as race, class, etc. Crenshaw (1989) first used the term intersectionality to describe Black women's dual subordination. The intersectionality metaphor applies familiar ideas about physical space in order to make the social identities like race, class, gender, etc. more tangible, which provides new angles of vision on each system of power, including how they diverge from one another (Collins, 2019).

Grande (2005) reminded researchers to be mindful that mainstream feminism dominated by white, middle-class women continues to serve privileged ethnopolitical interests and capital investments through discourses. Beddoes and Borrego (2011) argued that researchers should be aware of the feminist stance they take, that is, if they acknowledge power relations and historical oppression when they advocate for the women. Without this awareness, they risk continuing the very conditions they seek to change.

Based on this investigation of theorizing feminism, I next explore key concepts often associated with feminist theory: power, objectivity, reflexivity, and outsider within status. Through elaborating these concepts, I seek to conceptualize the feminist theory that informs this study.

Power

McCann and Kim (2013) argued that power is the most important concept in feminist theory because the purpose of feminist theory is to "inform effective politics" and "be accountable to politics" (p. 1). Feminist research builds on understanding issues, power, and knowledge and translates these insights into social research practices (Hesse-Biber & Piatelli, 2012).

The power dimension of gender was central to the concept of patriarchy and to the idea of men as a dominant sex class, i.e., men's power is over women, men are active, and women are passive in social relations (Connell & Pearse, 2009). Using a three-layered approach, Allen (2018) described power as a resource, power as domination, and power as empowerment. The first understanding of power is that power is "a positive social good, a resource, the distribution of which among men and women is currently unequal" (p. 7). The second view of power is as a relationship, specifically, the relationship between male domination and female subordination. From this perspective, feminist theory claims that women should have power equal to men, which challenges the domination system. In the third understanding of power, power is positioned as empowerment or transformation. According to this view, power "grows out of feminine traits, capacities, or practices" and tends to revise the masculinist conceptions of social and political life (p. 7).

Allen (2018) argued that, although each perspective of power has provided critical insights into women's experience, each has limitations and often emphasizes one aspect of multifaceted power relations. Building on these three perspectives (i.e., power as a resource, power as domination, and power as empowerment), Allen articulated a feminist conception of power that must be able to "make sense of masculine domination, feminine empowerment and resistance, and feminist solidarity and coalition-building" (p. 123). Further, Allen explained that domination "is in the particular kinds of power that men are able to exercise over women" (p. 123). Empowerment and resistance require that women respond to such domination; solidarity and coalition-building in power refer to what "feminists exercise with each other and with men in allied social movements" (p. 123). Keeping a feminist conception of power in mind, the commitment of feminist research is to challenge unequal distribution of power and oppression and contribute to social justice (Hesse-Biber & Piatelli, 2012).

Objectivity

Haraway (2016) said that "feminist objectivity means quite simply situated knowledges" (p. 354). By practicing strong objectivity, a researcher gains a more complex and theoretically rich set of explanations of the group's lives. Positivist epistemology is based on logic and empiricism. In the pursuit of knowledge building, researchers must be objective and value-free. Positivist objectivity is a bounded truth that lies in the social reality waiting to be discovered, leading to partial knowledge (Hesse-Biber, 2012). Feminist theory questions positivist objectivity, supposedly representing impersonal rules and ignoring the context of scientific theories that reflect masculine ideals and convey a stereotypic voice of authority (Morawski, 1997).

Harding's (1991) conception of "strong objectivity" is a specific example of practicing the basic premise of feminist objectivity. Strong objectivity challenges positivism's "weak" form of objectivity and tends to maximize knowledge for marginalized people rather than for only a dominant group. Strong objectivity demands that researchers examine both the research process and the cultural agendas and background assumptions of research's origins (Harding, 1991).

Feminist researchers start from women's lives to maximize objectivity and achieve strong objectivity. They challenge conventional accepted research norms, such that maximizing objectivity that aligns with situated knowledge systematically shaped by someone's social location (e.g., Harding, 1991; Wylie, 2012). A researcher obtains fuller insights into society as a whole and generates less distorted knowledge (Hesse-Biber, 2012). "These forms of knowledge allow for subjectivity between the knower and the known, rest in the women themselves (not in higher authorities) and are experienced directly in the world (not through abstractions)" (Collin, 2016, p. 310). Rather than separating from the situations, feminist researchers position themselves in situations where their lives affect the situations in which they live.

Reflexivity

Aligning with strong objectivity, feminist research projects also require reflexivity, which depends on a researcher positioning herself at the same investigation level as her participants. In doing so, a researcher gazes back at her own cultural values and interests in conducting the research project and examines the cultural assumptions and implications that situated it (Harding, 1991). Hesse-Bibber and Piatelli (2012) explained two central missions of reflexivity: self-critical action and a communal process. Through self-critical action, researchers explore how their theoretical positions and biographies shape what they choose to study and their approach to studying it. Reflexivity is also a communal process that helps a researcher foster sharing and engaged relationships, producing less hierarchical and more ethical, socially relevant research.

Reflexivity can help both the researcher and the participants develop critical consciousness, which can also improve stakeholders' lives during the research and transform relationships among the researcher and the participants (Maguire, 1987). During an inquiry, a researcher and a participant develop reflexive relationships. A researcher experiences a moment or moments during a conversation when an exchange of roles occurs—the researcher as a participant and the participant as a researcher. "By reflexively listening and engaging in dialogical relationships with our participants, we can bring the blind field into view and radically change the way we know and what we know." (Hesse-Bibber & Piatelli, 2012; p. 567). Thus, reflexivity allows a researcher to see herself through the eyes of participants. By openly sharing background, commitment, and research interests, a researcher works through power imbalances, which leads to a greater understanding of participants.

Outsider Within

Collins (1986) introduced the term *outsider within* to describe researchers from marginalized social groups crossing disciplines. She argued that an *outsider within* position provides a unique lens for Black feminists to develop a particular standpoint on the relations among self, family, and society. Black women are not the only *outsiders within*; many marginalized people moving into a community that historically excluded them could experience *outsider within* status, though they engage in the involvement and insider position in some communities. Collins (2016) wrote that "outsiders within occupy a special place—they become different people, and their difference sensitizes them to patterns that may be more difficult for established sociological insiders to see" (p. 312). The new status of outsiders within generates tensions that they are required to resolve. Some outsiders within suppress their difference and adjust their thinking as normal insiders while others remain outsiders. Either choice could alleviate the tensions but also could weaken diversity and blur boundaries among disciplines.

The best balance is that outsiders within embrace the tension and bring their own personal and cultural experiences into the community. As a researcher, I come into this dissertation study and continue experiencing *outsider within* status in communities of different cultures, languages, and disciplines. The *outsider within* status offers me a unique perspective to observe my participant and reflect on our interactions.

Feminist Theory in Mathematics Education

In her book *In a Different Voice,* Gilligan (1982) listened to both men and women resolving dilemmas in their lives. Gilligan used the terms *separation* and *connection* to describe two conceptions or experiences of self. Gilligan proposed that, because development has been premised on separation and told as a narrative of failed relationships, the ways girls develop do not fit the categories if relationships are derived from male experiences. This perspective calls attention to the assumption about connections that have informed the accounts of human development. Gilligan argued that women's voices should be included conversations about human development.

Belenky et al. (1986) adopted Gilligan's (1982) concepts of *separate* and *connected* knowing and posited two contrasting categories of procedural knowing. Separate knowing excludes a possible truth, while connected knowing builds on exchanging personal experiences. Based on the assumption that connected knowing is women's preference and could enhance the education of women, Becker (1995) addressed the implication of women's ways of knowing in mathematics, which was that information should not be ignored. In so doing, teaching mathematics should respect and accept learners' way of knowing and build instruction on learners' learning traits.

Women and Mathematics

Building on Belenky et al.'s (1986) five ways of women's knowing, Kaiser and Rogers (1995) described five phases between women and mathematics in historical contexts, specifically in the landscape of Western culture.

Phase 1: womanless mathematics;

Phase 2: women in mathematics;

Phase 3: women as a problem in mathematics;

Phase 4: women as central to mathematics; and

Phase 5: mathematics reconstructed (p. 3).

In Phase 1, doing mathematics is a masculine trait. Mathematical characters were referred to as "him" or "he." Although many women pursued mathematics, they did not believe in their own experiences and relied on an authority for all knowledge without question (Becker, 1995). Phase 2 began in North America in the 1970s, when a few women mathematicians were recognized but treated as exceptions and viewed as more like men. This culture suggested that successful women in mathematics lived with the contradiction of mathematics and femininity (Nosek et al., 2002; Walkerdine, 1998). Phase 3 began in the early 1980s when gender reform of mathematics stimulated intervention projects with the explicit goal to increase girls and women's interest in mathematics. However, the interventions mostly aimed to fix girls' interests, whereas mathematics itself was not questioned (Boaler, 1997). Phase 4 aims to disclose privilege and redistribute power in mathematics education. This phase emphasizes collaboration over competition and diversity over monotony. In this phase, feminist scholars fight for a new epistemology that brings justice to women in mathematics education (Kaiser & Rogers, 1995). In Phase 5, mathematics and its teaching are reconstructed involving a fundamental shift in what counts as mathematics, how to teach it, and the relationship of mathematics with the surrounding world. The literature reviewed suggests that Phase 3 remains at a practical level; Phase 4 is still in a theoretical stage; Phase 5 has not been attained.

Feminist Studies in Mathematics Education

Inspired by the early feminist perspective, scholars in mathematics education have opened a conversation about feminism and mathematics (Becker, 1995; Burton, 1995; Damarin, 1995;

Fennema & Hart, 1994). Ernest (1991) noted, “the cultural domination of rational and scientific knowledge by masculine values, serves to legitimate and sustain men’s domination of the power, status, and wealth, and hence political hierarchies in society” (p. 278).

Since the 1980s, studies have documented feminist theories in science and science education (Brickhouse, 2001). However, little attention has been given to a feminist view of mathematics (Turner, 1995). Building on feminist research in science, Burton (1995) claimed that feminist epistemology challenges the traditional notion of mathematics and questions what mathematics is and how mathematics is learned, which opens mathematics to different communities. Burton proposed feminist knowing of mathematics regarding its “person- and cultural/social-relatedness; the aesthetics of mathematical thinking it invokes; its nurturing of intuition and insight; its recognition and celebration of different approaches, particularly in styles of thinking; and the globality of its applications” (p. 287). Given this definition, knowing mathematics is related to who is claiming knowing and how that knowing is being presented.

Further, Fennema and Hart (1994) argued that a feminist perspective could contribute to mathematics education regarding what research questions are explored, whose voice is heard, and which methods are employed. Damarin (1995) also asserted that a feminist standpoint in mathematics education could make mathematics more accessible, enable changes in mathematics and its teaching, thus potentially improving women’s mathematics situation.

Nevertheless, few researchers have conducted feminist research in mathematics education. Feminist mathematics research and learning environment implies that power is redefined, teachers facilitate learning, and learners construct their knowledge rather than accept it from authorities (Jacobs, 2010). Rather than creating an instructional setting, a feminist learning environment aims to promote equity and power-sharing, encourage cooperation and collaboration, value intuition and emotion, and allow room for authorship and agency in learning (Anderson, 2005). Drawing on feminist and post-structural theories, Walshaw (2001) contended that post-structural analysis in feminist research offers a different story of girls’ mathematics experiences in school, which provides a transformative opportunity traditionally closed to girls. However, feminist analysis is largely absent in mathematics education. Damarin (2008) pointed out “the boundary separating mathematics from women’s studies and feminist theory, while not as forbidding as a prison wall, is nonetheless substantial and rarely crossed” (p. 101). Further, she critiqued the ambiguity of the discourse in mathematics education research. The term *gender* indicates women, girls, and feminine in a research agenda, and *gender and mathematics* and *women and mathematics* are treated as equivalent, which causes discursive barriers of research on women in mathematics.

My inquiry builds on previous feminist research in mathematics education, aims to cross the boundary between mathematics and women's studies, and further extends the feminist literature in mathematics education. Through reviewing the feminist analyses from literature, I have gained a better understanding of using a feminist lens to analyze women's mathematics and non-mathematics experiences.

The Affordances of Theoretical Perspectives

These theoretical perspectives (i.e., Akkerman & Bakker, 2011; Collins, 2016; Harding, 1991; Hesse-Bibber & Piatelli, 2012) allowed me to examine the lower participation of women in mathematics-related careers and support informal learning environments as a promising practice to promote women in mathematics. Through a narrative inquiry approach, I explored the lived experiences of two female educators who engage in informal STEM practices. Specifically, I have worked alongside Laura for three years. We have developed a collaborative relationship and bring our own expertise to our collective experiences. Throughout the collaboration, both of us have crossed multiple boundaries at interpersonal and intrapersonal levels (Akkerman & Bruining, 2016). For example, as we engaged in developing the informal STEM program, Laura brought her practitioner viewpoint, and I contributed a researcher's perspective. In our interactions, we recognized each other's viewpoints first; based on that, we exchanged information and valued each other's insights; finally, we understood each other's perspectives and collaborated to solve shared problems. The boundary crossing perspective provided me a lens to understand the experiences in crossing boundaries and understand the continuities and discontinuities at boundaries. Feminist theory calls attention to people who have been left behind in mainstream research and values the perspectives and lived experiences of women as knowledge sources. In this dissertation study, feminist theory and boundary crossing are merged as a unified theoretical framework that provides critical and analytic insights.

Both Feminist theory and the boundary crossing perspective acknowledge that knowledge is a social construction, and that learning is about engaging in multiple social activities and practices. Feminist theory arises from the varied communities to which women belong, which may differ according to race, class, and sexual orientation, and other social identities. Women from diverse ethnic backgrounds may also maintain different perspectives. For instance, the outsider within status can be a source of frustration and creativity and lead to unique perspectives and insights (Collins, 2016). Similarly, the boundary crossing perspective contends that people, as part of communities of practice, inhabit many different communities at once, engage in different practices and gain different perspectives (Wenger, 1998).

Feminist theory and boundary crossing have different emphases. Feminist theory is building on the assumption that most people, including most women, live in male-dominated societies; dominant systems of knowledge and the structure of everyday life distort their perceptions (Jagger, 2016). From this viewpoint, feminist theory focuses on social change rather than only gaining understanding. Conversely, rather than promoting changes, learning at boundary crossing focuses on understanding experiences within boundaries and uses boundary objects to encourage learning (Akkerman & Bakker, 2011). In this sense, feminist researchers could learn from the boundary crossing perspective that the feminist struggles and conflicts could be mediated through crossing boundaries of different practices. The feminist perspective is useful to learning in boundaries research on resolving the tension on gender issues in learning, such as gender stereotypes in mathematics. Meanwhile, the boundary crossing perspective could mediate tensions created by the outsider within status.

In summary, boundary crossing research in education has made significant contributions in teaching and learning but has not specifically addressed women's issues, while feminist theory advocates for women but is not often focused on learning. Although differences exist between them, the theories complement and align with one another. The compatibility accounts for the complexity of my research focus on women, mathematics/STEM, and informal learning.

Chapter Summary

In this chapter, I synthesized relevant literature related to women in mathematics/STEM, the nature of mathematics education, informal education, and the theoretical framework, including the boundary crossing perspective and feminist theory, shaping the proposed study, and informing data analysis. The boundary crossing perspective provides an analytical lens to understand both Laura and my experiences in crossing boundaries between formal and informal learning, between mathematics and STEM, and between practitioner and researcher. Feminist theory brings women's experiences into the center of inquiry. Inextricably bound with equity and social justice, feminist theory uncovers women’s historical mathematics/STEM participation and calls attention to women’s viewpoints of mathematics.

CHAPTER 3. NARRATIVE INQUIRY METHODOLOGY

Informed by the theoretical frameworks described in Chapter 2, I utilize a narrative inquiry methodology, which relies on storytelling to understand participants' lived experiences (Connelly & Clandinin, 1990). As a qualitative methodology, narrative inquiry carries various meanings in oral, written, and visual forms, with the goals of understanding people's experiences, constructing individual identities and forming communication channels (Chase, 2018; Clandinin & Connelly, 2000; Riessman, 2008). With the goal of building a reciprocal and collaborative relationship with Laura, narrative inquiry enabled me to co-construct the inquiry with her. In this chapter, I first present the definitions and descriptions of narrative inquiry that influence my thoughts about narrative inquiry. Then I describe the conceptualization of narrative inquiry used in this study. Later, I delineate the process of becoming a narrative inquirer. Following that, I summarize the narrative inquiry methodology used in mathematics/STEM education and narratives of teacher knowledge. In the research methods, I describe participants, data collection, and data analysis. Lastly, I discuss the limitations of the study.

Defining Narrative Inquiry

In this section, I review definitions of narrative inquiry used across several disciplines. Rather than comparing its uses in different disciplines, I highlight the important influences that have helped me conceptualize narrative inquiry for my purposes.

Bruner (1986) stated that narrative, as a mode of knowing, "deals in human and human-like intention and action and the vicissitudes and consequences that mark their course" (p. 13). According to Bruner, an individual's working intelligence should be understood by considering all interactions within their professional and social networks. The narrative thus offers a way of analyzing human events and the construction of reality. Meanwhile, narrative inquiry fulfills the dual roles of both representing and constituting reality (Bruner, 1991). Drawing on Bruner's narrative mode of knowing, Polkinghorne (1995) explicated narrative as "the type of discourse composition that draws together diverse events, happenings, and actions of human lives into thematically unified goal-directed processes" (p. 5). The narrative explains human actions by configuring the various elements into an organic whole in which each element is connected to the central aim of the action (Polkinghorne, 1995). The narrative thus makes a particular incident understandable by a series of descriptions and explanations. Beyond focusing on personal stories, narrative inquiry reveals truths about the human experience and constructs individual identity and meaning (Riessman, 2008).

Dewey (1938) conceptualized experiences as individuals' interaction with social contexts, which are temporal, growing out of previous experiences and leading to future experiences. Building on Dewey's (1938) experience theory, Clandinin and Connelly (2000) rooted narrative inquiry in education. They gave a working definition of narrative as "a way of understanding experience. It is a collaboration between researcher and participants, over time, in a place or series of places, and social interaction with milieus" (p. 20). The authors stressed the researchers' subjectivity and highlighted relationships between researchers and participants. According to Clandinin and Connelly (2000), an individual's experiences provide a way to understand their education. They also offer a more detailed and informative way to learn about the general construct of continuity of the individual's life. Following Clandinin and Connelly's suggestion, I engage in narrative inquiry research by embracing narratives as both the method and phenomena of study.

Clandinin and Connelly (2000) described a three-dimensional narrative inquiry space, wherein narrative inquirers move backwards and forwards, inwards and outwards, and locate themselves in space. These three dimensions are temporal, which relates to time, personal and social interactions, and place(s). Working within the three-dimensional narrative inquiry, a narrative inquirer has an opportunity to attend to a participant's life, inquire into the participant's experiences, their own experiences, and the co-constructed experiences (Clandinin, 2006). Thus, a researcher can address both personal and social issues by looking inward and outward and addressing temporal issues by looking at the event, reaching back into its past, and looking forward to its future (Clandinin & Connelly, 2000).

Chase (2018) defined narrative inquiry more comprehensively:

> A personal narrative is a distinct form of communication. It is meaning-making through the shaping of experience; a way of understanding one's own or others' actions; of organizing events, objects, feelings, or thoughts in relation to each other; of connecting and seeing the consequences of actions, events, feelings, or thoughts over time (in the past, present, and/or future) (p. 951).

Chase explained that *communication* refers to the narrative, including visual, oral, and written forms. *Feelings* or *thoughts* indicate that narratives are about the events and an individual's reflections on those events. This definition notes time as being "in the past, present, and/or future," which resonates with the continuity characteristic of Dewey's (1938) theory of experience.

Conceptualizing Narrative Inquiry

Drawing on the definitions of narrative inquiry described above, I ground my understanding in Dewey's (1938) theory of experience and Clandinin and Connelly's (2000) three-dimensional narrative inquiry space. Dewey theorized that one principle of experience is both social and personal; people are individuals who need to be understood as such. Meanwhile, individuals live in relationships with others and multiple social contexts; people need to be understood in social relations contexts as well. Clandinin and Connelly (2000) interpreted the social and personal aspects of experiences as both inward and outward, reflecting the interaction between internal conditions and external environments. Following these conceptions of inward and outward, narratives offer me a new perspective on the inner world of individuals through an exploration of their experiences. This opens channels for learning about the participant's life and experienced reality. In other words, narrative inquiry gives me access to my participant's identities and personality. By constructing social meanings, narrative inquiry enables me to go beyond the stories, and capture personal insights and ideas that may address social concerns, such as how life experiences influence identities within different social contexts.

Situation is a principle of experience (Dewey, 1938). Clandinin and Connelly (2000) conceptualized the situation as a physical landscape, or setting, with topological boundaries. In this study, the interaction between Laura and me occurred through GEMS, which is not a specific location or physical space, but an amorphous situation. GEMS has different divisions, including GEMS clubs, a GEMS research team, and a GEMS development team. Some groups have existed for more than 20 years; others emerged when GEMS transferred to Purdue; still others started more recently.

Since the Fall of 2018, I have been involved in several GEMS groups with Laura and others (e.g., faculty, graduate students, undergraduate students). I view these groups as communities that have different goals and overlapping membership. Most of the time, we meet online through videoconferencing and communicate via email and phone calls. In my study, GEMS, as a situation, allows me to establish a landscape in which Laura's stories and experiences are investigated, and I can unpack the complexities of the relationship between Laura and me (Clandinin & Connelly, 2000).

Continuity is another principle of experience (Dewey, 1938), which indicates that a current experience is influenced by past experiences and will influence future experiences. Clandinin and Connelly (2000) used the terms *forwards* and *backwards* to describe the temporality of experiences. During the inquiry into Laura's experiences, both of us recalled past experiences and reflected on present experiences. Moving backwards and forwards, I had the opportunity to listen to my inner voice, which helped locate my temporal positionality in the inquiry process.

In summary, narrative inquiry provided a three-dimensional inquiry space that allowed me to reach inward to the collaborative relationship with the participant. Moreover, a narrative inquiry enabled me to move outwards to cross boundaries between different social contexts to gain new experiences, which shaped my identities and understanding. Looking backwards, Laura and I both had the opportunity to reframe our own experiences. Looking forwards offered me a vision of the study's contributions to our communities and society.

Developing as a Narrative Inquirer

Connelly and Clandinin (1990) described narrative inquiry as occurring within the relationship between the researcher and the participant. Both researchers and participants tell stories of the research relationship. Ideally, they construct and empower a caring, equal community within which the researcher and the participants see themselves. In beginning a narrative inquiry, a researcher must be aware of constructing a relationship in which both voices are heard. In their later work, Clandinin and Connelly (2000) described the development of narrative inquirers. They suggested that to become a narrative inquirer, a researcher needs to walk amid the stories they will tell:

> Narrative inquirers settle in, live and work alongside participants, and come to experience not only what can be seen and talked about directly but also the things not said and not done that shape the narrative structure of their observations and their talking (p. 68).

Because of the collaborative work relationship between Laura and me, we interact as real persons rather than the researcher and the researched. The ideas, thoughts, and feelings flow during the conversation and non-conversational interactions.

As I explored narrative inquiry methodology, I experienced the four moves that Pinnegar and Daynes (2007) proposed and developed toward being a narrative inquirer. The four turns are clear indicators of movement toward narrative inquiry, both in the research lives of individuals and in the discipline. These include (1) a change in the relationship between the person conducting the research and the person participating as the subject, (2) a move from the use of numbers to the use of words as data, (3) a change from a focus on the general and universal toward the local and specific, and (4) a widening in acceptance of alternative epistemologies or ways of knowing. Below, I describe my understanding of those turns in detail in the context of this study.

For the first turn, Pinnegar and Daynes (2007) suggested that researchers shift from a neutral stance in the researcher-researched relationship toward a relational view. The environment in which the interaction between a researcher and a participant occurs will influence their interactions. A researcher not only understands the relationships between the humans involved in the inquiry and the interactions between humans and their environment, but also understands the researcher's emerging self and the participant in the inquiry. This turn echoes Clandinin and Connelly's (2000) suggestion that those who conduct narrative inquiries should be a researcher in the relationship and walk into and play a role in the participant's story.

Pinnegar and Daynes's (2007) second turn is from data consisting of assigned numbers to data consisting of words. This process does not mean a rejection of numerical data but a recognition that word data provides ways of holding meaning together in more complex, relational, and nuanced ways. Turning toward word data allowed both Laura and me to have more space to present an understanding of concepts under consideration.

The third turn is from the general to the particular (Pinnegar & Daynes, 2007). The local knowledge of particular social scientists investigating and responding to the individual problems of humans seems increasingly more productive (Polkinghorne, 1988). Turning to the particular allows researchers to engage people without "expertise," which provides authentic and powerful evidence of the need for political and social change. In this turn, a single topic and particular people involved in specific settings serve as a lens through which to see the entire context (Pinnegar & Daynes, 2007). For instance, Clandinin and Connelly (2000) described their own narrative inquiry examples with individual teachers and principals.

The fourth turn requires blurring the knowing. There are multiple ways of knowing and understanding human experiences (Pinnegar & Daynes, 2007). Narrative inquirers recognize the tentative and variable nature of knowledge. Aligning with feminist subjective knowledge, the acceptance of the relational and interactive nature of human research, the use of the story, and a focus on a careful accounting of the particular are hallmarks of knowing in narrative inquiry.

Pinnegar and Daynes (2007) used *turn* to highlight the movement towards fully embracing narrative inquiry. During the inquiry process, I was intentionally aware of the interactions between Laura and me, practiced using individual stories as data, and situated the stories in a specific context. My goal was to explore the complexity of boundary crossing experiences of both myself, a novice mathematics education researcher, and Laura, an informal STEM educator, coming together and engaging in a collaborative relationship. Narrative inquiry allows for capturing nuanced continuities and discontinuities at the boundaries.

Narrative Inquiry in Mathematics/STEM Education

Using a narrative inquiry methodology allows researchers to prompt participants' stories, enabling them to hear marginalized voices and see multiple teaching layers and various ways of learning mathematics. Foote and Bartell (2011) examined the life stories of a group of scholars in mathematics education who identified themselves as having a particular interest in and concern for issues of equity and diversity. Through exploring the scholars' life experiences, the authors confirmed that researchers' personal and professional experiences impact their positionality related to issues of both mathematics and equity in mathematics education scholarship. By analyzing teachers' life stories, researchers gain a contextualized and integrated understanding of teachers' prior experiences and how those experiences have shaped their teaching, knowledge, and mathematical identities (e.g., Drake & Sherin, 2006; Kaasila, 2007).

Narrative inquiry is an emerging approach to situate mathematical experiences and identities in sociocultural contexts. As such, equity and social justice issues can be foregrounded in mathematics education. In de Freitas's (2008) study, pre-service mathematics teachers wrote an autoethnography about their past and present experiences with school mathematics, focusing on specific emotional moments that left a strong imprint on their memories. These personal narratives helped the pre-service mathematics teachers become aware that their identities were established through mathematics education experiences. The author suggested that the use of an autoethnography of identity construction can help pre-service teachers understand that sociocultural issues frame their school mathematics experiences, which can better prepare them to teach for diversity.

Narrative inquiry highlights individual experiences and acknowledges the role of researcher in the research, which provides a new lens through which to view mathematics education. Drake and Sherin (2006) worked with two elementary teachers to help them implement a reformed mathematics curriculum and reflect on their teaching practices. They found that the dual roles (i.e., researcher and participant) enabled them to gain significant insights into the teachers' experiences and perceptions of curriculum in the form of anecdotes or stories, which came from chance meetings in the hallways or late-night phone calls.

Narrative inquiry provided a three-dimensional inquiry space to document the stories and experiences of the researcher and the participants, which allowed the researcher to enter the complex school environment and to understand the context in which teachers construct and enact their technology, pedagogy, and content knowledge. Boniface's (2020) doctoral dissertation used narrative inquiry to document her experiences with three teachers involved in developing and incorporating computer science into the elementary curriculum which offered a reference for me to explore the collaborative relationship between Laura and me.

To understand teachers' specific practices in the context of using reform-oriented mathematics curriculum, Drake (2006) explored the teachers' narratives of their learning and teaching mathematics experiences and analyzed their turning points stories. The author concluded that understanding teachers' identities and personal histories as learners and teachers of mathematics are the keys to both facilitating and understanding their implementation of the reform curriculum which provided an example for me to explore Laura's personal and professional experiences with mathematics with an aim to understand her fluid view of mathematics.

In working alongside a teacher in an elementary mathematics classroom, Ross (2003) observed the class, communicated with the parents, and interacted with the students. Through writing and analyzing field notes, she explored the mathematics curriculum from the perspectives of the teacher, social milieu, subject matter, and learner in the classroom context.

In another example of working with an individual participant, Flores (2017) explored a four-year collaboration with an eighth-grade mathematics teacher. In the study, Flores and the teacher both performed dual roles as researcher and participant. Flores found that using narrative inquiry allowed her to reflect on her own experiences as a teacher and a learner in mathematics;

also, as a narrative inquirer, she participated in the teacher's experiences in and out of school and across time, which provided her an opportunity to understand the teacher's personal knowledge in designing and implementing the curriculum.

Narrative inquiry has also been utilized in self-study in mathematics education. For example, Butler et al. (2019) studied a multidisciplinary collaborating team charged with improving mathematics pre-service teacher education. The narrative inquiry process enabled team members to explore, deepen, and sustain their connections with one another and their understanding of the shared focus underpinning their work as teacher educators. Some members reported that the inquiry allowed them to develop a greater capacity to listen and reflect on both their own communication and the communication of others. In another self-study, Smith (2006) used narrative inquiry to understand pre-service teachers' experiences as well as to theorize her own practice in a methods course. The course goal was to encourage pre-service teachers to become researchers of their own practice and authors of their own narratives of learning. On the one hand, pre-service teachers used narratives to revisit and reframe their experiences as teachers and learners and addressed issues that emerged in their theory into practice. On the other hand, the researcher constructed accounts of practices to describe aspects of pre-service teachers' narratives and document her own learning experiences in teaching the course.

STEM fields are often linked to personal and national economic prosperity, technology, and scientific innovation. However, there are disparities in STEM participation by gender, race, and ethnicity. Women and racial, gender, and socioeconomic minorities often participate less in STEM fields than their White male peers (National Center of Educational Statistics, 2019). The narrative inquiry approach focuses on individuals' lived experiences and can allow members of underprivileged groups to have a voice. In this way, the narrative is a means through which those who have been historically marginalized in STEM fields can be heard (Foor et al., 2007).

In STEM education, narrative inquiry often addresses critical issues, such as race, gender, or integration of race and gender. Kersey's (2018) dissertation study addressed transgender students' experiences and identities in mathematics and STEM fields. She found that, in working with a marginalized population, using narrative inquiry allowed for more subjectivity and richer details about the unique features of the participants' lives than other approaches. Gladney (2016) used a narrative inquiry approach to examine eight African-American women's lived experiences in their STEM careers through storytelling. In narratives of critical events, Gladney unpacked these women's challenges, barriers, and positive experiences related to their academic and professional career trajectories, including STEM pre-college preparation, professional career development, and promotion potential. By analyzing the participants' narratives of their experiences, she found that mentorship, professional career development, and training certification contributed to improving African-American women's participation in STEM fields.

The studies summarized above demonstrate the power of narrative in obtaining critical insights into teachers' experiences at personal and professional levels. In addition, these studies provided me with exemplars for working with individual participants and investigating critical issues related to women's participation in mathematics/STEM. Using narrative inquiry enabled me to establish a broader landscape looking into mathematics education to view mathematics and STEM, women and mathematics, and mathematics and its teaching. The narrative inquiry approach allowed both Laura and me to develop a greater capacity to listen and reflect on communication (Butler et al., 2019). In addition, storytelling allows me to cross cultures, languages, and the boundaries of disciplines to understand Laura's experiences and perceptions of mathematics and to obtain critical insights into women in mathematics.

Narratives of Teacher Knowledge

Elbaz (1981) conceptualized teacher knowledge as practical knowledge, "encompassing knowledge of practice as well as knowledge mediated by practice" (p. 46). Specifically, Elbaz described practical knowledge as situational, theoretical, personal, social, and experiential. The situational aspect of personal practical knowledge suggests that teacher knowledge is practical to respond to the various situations of teaching. The theoretical aspect of teacher knowledge moves toward generalizations across different areas in teacher education. The personal aspect indicates that "teachers use their knowledge to enable them to work in personally meaningful ways" (p. 49). The social aspect reveals that teacher knowledge is shaped and constrained by teachers' social conditions. The experiential aspect of teacher knowledge demonstrates that teachers' knowledge is embedded in their personal and professional experiences.

Building on Elbaz's (1981) concept of personal practical knowledge, Connelly and Clandinin (1988, 2006) expanded personal practical knowledge to include thinking narratively and valuing experiential and subjective knowledge. Clandinin (1985) explained that personal practical knowledge is not just content knowledge or structure knowledge. Rather, "personal practical knowledge is revealed through interpretation of observed practices over time and is given biographical, personal meaning through reconstructions of the teacher's narratives of experience. It is knowledge that is experiential, embodied and based on narrative of experience" (p. 363). Petra Munro (1998) retold the life stories of three female teachers in the book *Subject to Fiction*. Crossing the life narratives of the three women teachers, a central theme that emerged was *becoming a teacher*. The resistance to entering teaching was interpreted as a conflict between gender ideologies and their understanding of self which later contributed to their personal practical knowledge as teachers (Munro, 1998).

Connelly and Clandinin continued to develop the concept of personal practical knowledge through research collaboration with teachers and schools (e.g., Clandinin & Connelly, 2000). Their studies delineated that personal practical knowledge is imbued with a person's experiential history. Personal practical knowledge, therefore, needs to be understood in terms of individuals' histories, both professional and personal. In the collaboration between researcher/teacher educator and teacher, valuing personal histories and narrative experience as a mode of teacher knowledge is a way to trust, respect, and support the teacher as knowing and knowledgeable (Connelly & Clandinin, 1988).

Ross and Chan (2016) argued that personal practical knowledge provides an alternative way of understanding the role of teachers in curriculum choices and curriculum development. In addition, personal practical knowledge challenges the emphasis on teachers' content knowledge, teaching, and other traditional approaches to teacher knowledge and neglects what the teacher knows about teaching and how the teacher's knowledge has been developed. Conceptualizing knowing as personal practical knowledge enables researchers to better understand that "teachers are simultaneously drawing on all aspects of their knowing" (p. 10) in planning and implementing teaching. Then, the researcher/teacher educator can provide responsive support to teachers in collaboration and professional development. Therefore, studies on teacher knowledge should "be more carefully positioned to examine the tensions between teachers' knowledge (as it exists and as it develops) and the contexts in which teachers work" (p. 5).

Teachers' personal practical knowledge is embedded in their personal, professional, and social contexts, which are intertwined with their fluid, evolving identities (Clandinin & Connelly, 2000). Phillion (2002) used a narrative approach to capture the day-to-day real life of her participant, Pam, a classroom teacher, which she termed as *narrative multiculturalism* to indicate a fluid, evolving, transforming, and changing practical knowledge. He (2002) used *in-between* to describe ambivalence and ambiguity of experiences of three female teachers, including herself, crossing cultures and languages between Eastern and Western worlds. Their teacher knowledge was developed from "cross-temporal, cross-historical, cross-cultural, and cross-generational aspects of our [their] daily living" (p. 333).

In her book *Personal Narratives of Teacher Knowledge: Crossing Cultures, Crossing Identities*, Betty Eng (2021) reconstructed her cross-cultural experiences and reflected on how these experiences have shaped her personal practical knowledge as a teacher educator. By telling and retelling stories of her experiences, Eng explored her culture and identity in the crossing and crisscrossing of cultures and communities. Unpacking the complexity of her identity development in multiple historical contexts, Eng expressed that teacher knowledge is individual, experiential, and subjectively shaped by context.

Previous studies have provided me with the paradigm of using a narrative inquiry approach to explore teachers' narrative histories to understand their teacher knowledge and identities across contexts (e.g., Chen et al., 2017; Eng, 2021; He, 2002). As an informal educator, Laura's knowledge is not content-focused; instead, her knowledge is rooted in practice and experiences in and beyond GEMS. Exploring Laura's personal and professional experiences gave me an opportunity to value her experiences as a source of her personal practical knowledge. Laura's personal and professional narratives informed her teacher knowledge in developing GEMS, choosing a curriculum for GEMS, and supporting other informal educators. Exploring Laura's personal history of becoming a woman, teacher, and educator inspired me to reflect on my own personal and professional experiences that guided me to become the person I am now.

In this narrative inquiry, I crossed multiple boundaries (linguistic, cultural, and disciplinary) and immersed myself in collaboration with Laura. My identity is intertwined with Laura's identity as a woman, teacher, and educator (Clandinin et al., 2009). Exploring these identities allowed me to look backward, forward, inward, and outward to locate our experiences in the social contexts and to interrogate the meanings of the experiences. As I embraced the fluid and evolving personal practical knowledge, I conducted this inquiry into our development of multiple identities in the past, present, and future.

Research Methods

In this section, I illuminate the research methods in the study. First, I introduce my primary participant, Laura, and describe my dual roles as both researcher and participant. Next, I explain the data collection and the data sources including various forms of field texts, interviews, and other materials and documents. Lastly, I explicate the data analysis process from transferring field texts to research texts to listening to different voices, then to the narrative analysis and analysis of narrative approaches.

Participants

Laura

Laura, the primary participant in the study, started the first GEMS club in 1994 for her daughter and other girls. When she heard her daughter complain "Math is hard," Laura searched research reports and found that many girls had a negative attitude about mathematics and science; they did not want to take higher-level mathematics classes and often had little confidence in mathematics and science. The findings from her investigation resonated with Laura as a parent. She realized that her daughter, as well as many other girls, struggled in these subjects. This realization motivated her to start an informal learning experience just for girls. Laura collaborated with her daughter's teacher to start the first GEMS club in the school. Since then, she has led mathematics and science activities in the GEMS club, applied for funding to support GEMS, and provided instruction for other GEMS leaders. Laura has visited schools in her county and has introduced her experiences to teachers and parents, encouraging them to also lead GEMS clubs.

In 2005, Laura built the first GEMS website on which she published collected resources and activities, making it a free online resource library for the public. From that point on, GEMS began to spread beyond her county. In 2011, Laura was invited to the Clinton Global Initiative Conference where she had the opportunity to meet many people who were working to increase involvement of women in STEM. In the summer following the conference, Laura developed the first GEMS Toolkit (later called the GEMS Handbook) and published it on the website. The GEMS Handbook includes the mission of GEMS, advice and instructions for starting a GEMS club, and sample activities. The GEMS Handbook played a significant role in spreading GEMS clubs across and beyond the United States.

For Laura, GEMS is like a seed that she planted; she cultivates it and takes care of it in her daily life. With Laura's day-to-day nurture and care, GEMS, like an experiential seed, has sprouted, flourished, and spread across the country. After retiring from her full-time job as a technology specialist, Laura began to devote more time and effort to the development and implementation of GEMS. She realized that no one else was as familiar with GEMS as she was, so she began to consider who could be a trusted successor to take care of GEMS. Finding a home for GEMS became an emergent work for Laura.

Purdue University entered Laura's mind after talking with a friend on the faculty. As an alumna, Laura trusted that Purdue, with its foresight and educational goals, could support GEMS' growth and development. In the spring of 2018, Laura decided to bequeath GEMS to Purdue University. A Purdue GEMS team, consisting of graduate students (including me), undergraduate students, and faculty, formed to begin work on the transition. As a core team member, Laura was invited to join the GEMS group via weekly online meetings to collaborate with us in further developing GEMS and conducting research related to various aspects of GEMS.

In addition to her identity as the founder of GEMS, Laura is the mother of two daughters, a former special education teacher, a technology specialist, and a writer. Laura wrote two works of historical fiction about her family history, *The All-Wise Being* and *Eel River Rising*, to memorialize family stories in their historical contexts. She has written children's books for her grandchildren, as well as a picture book, *Herndon: Then and Now*, using photos to record local history. In her spare time, Laura likes to quilt and videorecord her school and family to preserve history. In subsequence chapters of this dissertation, I invite readers to get to know Laura and learn about her experiences. Note that, in this study, I often use plural pronouns *we*, *us*, and *our*, to refer to Laura and myself.

My Role as Researcher and Participant

I am both a researcher and a participant in the study. Since the fall of 2018, while collaborating with Laura, I have participated in the GEMS group in three primary ways. As a team member, I serve on the GEMS development, marketing, and research teams. From a researcher's viewpoint, as I work alongside Laura and interact with her in these contexts, I learn about GEMS as an informal STEM learning environment in the past, present, and future (Clandinin & Connelly, 2000). I also observe Laura, who identifies herself as an outsider to the mathematics education community, collaborate with mathematics educators and develop expanded views of what it means to do mathematics. From a participant position, I have been crossing multiple boundaries (Akkerman & Bakker, 2011), including between mathematics education and STEM, and have developed a broader view of mathematics in STEM contexts. Also, I have crossed the boundary between the mathematics education community and informal education community. Over time, through collaboration with Laura in GEMS, I began to embrace the *outsider within* status (Collins, 2016) and to learn and change in *in-between* spaces (He, 2002).

Narrative Data Collection

Because of the nature of narrative inquiry, this study's data are co-constructed between Laura and me. In this study, field texts included various forms: interviews, field notes, family stories, autobiographical writing, documents, conversations, emails, and photographs (Clandinin & Connelly, 2000). The field notes and interviews were two primary forms of data. In the field notes, I record ongoing interactions between Laura and me, as well as my reflections, thoughts, and feelings in particular moments. I conducted two interviews to understand Laura's life story, which focused on her past experiences with GEMS. The third interview with Laura focused on her view of mathematics and her collaborative experiences within the GEMS group. A variety of other sources include GEMS documents, interviews with OGGs and Ms. Cooper, and GEMS meeting notes also served as important data.

Field Notes

Clandinin and Connelly (2000) suggested that in narrative inquiry the researcher must be a real person who records, in writing, their thoughts, feelings, and ideas at the moment. Because the collaboration between Laura and me is an ongoing process, writing field texts on what I have observed, experienced, and reflected on has helped me maintain an intimate relationship and establish a relational distance when I need to switch to a researcher role.

In addition to writing field notes, sometimes we recorded the conversation and discussion during the GEMS meetings. Later, I transferred the audio recordings into text, in which I added my interpretations or reflections. I also recorded meaningful informal conversations as field notes. For instance, Laura and I discussed a spatial activity over a phone call, which was informative regarding the development of spatial reasoning curriculum. During the call, Laura shared her insights about spatial skills. Afterward, I wrote notes in my own words to describe Laura's views of spatial reasoning curriculum.

In Fall 2021, Laura applied for a grant to support her purchase of an engineering-based STEM curriculum. She implemented the curriculum in a GEMS club with two groups of seventh grade girls. Through an online meeting platform, I observed Laura's teaching in her GEMS club and sometimes we debriefed evolving questions and issues during the sessions. I wrote field notes to record my observations and reflections and our discussions.

In addition, I used field notes to document my exploration of the theoretical framework. For me, exploring and fully embracing the theoretical framework was a long journey. I recorded the process in field texts, which helped me recognize how our interactions have shaped my theoretical framework and how the theoretical framework has impacted our interactions.

Clandinin and Connelly (2000) suggested that a researcher's dual role produces a dual quality of notes. Writing field texts has allowed me to remain alert to what participants say and do and what the researcher's field experiences are within the study. As I continued writing and composing field texts, I consciously reflected on my dual roles. Reading field notes allowed me to move back and forth between being fully involved with collaboration and being distanced from it (Clandinin & Connelly, 2000).

Interviews

As narratives are fluid and unexpected, a narrative interview often lacks the structure of a fixed interview format, encourages equality, and brings uncertainty into the conversation. Riessman (2008) suggested that the interviewer's role in narrative inquiry is to facilitate storytelling rather than to obtain answers to fixed-response questions, because in interviews, storytelling can occur at the most unexpected times. Kim (2016) suggested that a narrative interview should invite participants to speak in their own voices, express themselves freely, and create their own narrative schema. Riessman also noted that encouraging participants to speak in their own way can balance authority. Keeping these suggestions in mind, I conducted three interviews with Laura. The interview protocols evolved in the collaboration based on topics we had discussed that I wanted to explore further. I shared each interview protocol with Laura before the interviews. Appendix A contains the final interview protocols used.

In August 2019, I conducted the first narrative interview with Laura, focusing on her life story (Atkinson, 2007). At that time, although I had engaged in weekly GEMS meetings for two semesters, worked on organizing the GEMS website (https://gems.education.purdue.edu), and explored literature about informal STEM learning, I was still in a peripheral position on the GEMS team, an outsider to informal education communities, and puzzled as to why Laura started GEMS and why GEMS was needed. The interview provided me with an opportunity to understand Laura's life story and became the first step in exploring my questions about GEMS.

During the interview, Laura told "a fairly complete narrating of one's [her] entire experience of life as a whole, highlighting the most important aspects" (Atkinson, 1998, p. 8). In the interview, Laura chronologically described, in the form of stories, her experiences learning mathematics, as a parent of a mathematics learner, and as the founder of GEMS. I facilitated the storytelling by asking necessary questions (Kim, 2016), focusing on specific events and happenings that had left a strong imprint on her memories. The interview lasted 90 minutes, was audio-recorded, and was later transcribed by a professional transcriber.

In October 2019, a follow-up interview was conducted (a) to clarify information from the first interview; (b) to focus on the GEMS development trajectory, from the first club to the current phase; and (c) to look forward to the future GEMS program. This interview lasted 60 minutes, was audio-recorded, and was later transcribed using an online transcription tool.

The third interview was conducted in June 2021, after three years of collaboration between Laura and the Purdue GEMS team. Unlike the previous two interviews focusing on Laura's past experiences, this interview focused on Laura's reflective experiences in the collaboration. Specifically, the interview was intended to prompt her views on mathematics, informal learning, and the future GEMS program. Laura and I co-constructed this interview to address the important issues for both of us (i.e., women in mathematics/STEM). The interview lasted 90 minutes and was audio-recorded and later transcribed using an online transcription tool.

Other Data Sources

In addition to field notes and interviews, a variety of other data sources served to substantiate and supplement my inquiry into past, current, and future experiences that informed our personal practical knowledge. To inquire about my own personal narratives, I drew informational materials from my diaries, reflections, family stories, and conversations with my parents, students, and colleagues that facilitated my process of remembering and informing the stories of experiences. Laura provided her published and unpublished journals, her historical fiction, reports and documents about GEMS, and GEMS videos.

The GEMS website (https://gems.education.purdue.edu) as an online representation of GEMS and the GEMS Handbook (2019) which consists of the mission of GEMS, advice and instructions for starting a GEMS club, and sample activities, provided primary data sources for me to access GEMS and learn about GEMS. Laura's multiple publications about her teaching and leading the GEMS club (e.g., Jones, 2001, 2005, 2008) served as important data sources to learn about Laura's philosophy as an educator. Specifically, in her unpublished journal (Jones, 2002a), Laura reported on her two action research studies. The first study was conducted in 1997 to examine the impact of GEMS on the original GEMS girls' (OGGs) attitudes toward mathematics and science, Laura surveyed the OGGs and interviewed them as well as their classroom teachers and some parents. The second study was conducted in 2002 to investigate the OGGs' course taking and career interests. Laura surveyed the OGGs and interviewed the same group of parents. This journal (i.e., Jones, 2002) provided an important original reference for me to learn about the first GEMS club and Laura's experiences with this club.

In 2020, with Laura's assistance, the Purdue GEMS research team surveyed 14 OGGs and interviewed nine of them to understand the long-term impact of GEMS on their views of mathematics and their career choices. The data provided a retrospective perspective for me to perceive the first GEMS club. In addition, in 2020, Laura connected me to with Janet's teacher, Ms. Cooper, who co-led the first GEMS club with Laura. I interviewed Ms. Cooper via phone call which offered important information to understand GEMS from the perspective of a leader as well as a classroom teacher.

GEMS meeting notes, which documented our decisions and ideas in the moment, were used to understand our fluid and evolving experiences and identities. Emails and texts were also important forms of field texts; Laura and I exchanged ideas about research, the website, and activities through emails and texts. The two historical fictions, *The All-Wise Being* and *Eel River Rising*, Laura wrote, were used as supplement data sources to understand Laura's viewpoint of women's social position in her context. Additionally, I drew data from the OGGs' surveys and interviews and conversations with other GEMS leaders to enrich the narratives of GEMS.

Narrative Data Analysis

Narrative data are dynamic and influenced by contextual factors and the interaction of the researcher and the participant. Typically, the quantity of narrative data is quite large, and a researcher can take many directions to analyze data. Narrative researchers thus need to be comfortable with ambiguity (Lieblich et al., 1998). In this narrative inquiry, the *in-between* (He, 2002) situation and the *outsider within* (Collins, 2006) status triggered ambivalence and ambiguity in the data analysis. For instance, since the beginning of this inquiry, I have consistently thought about the position of mathematics in this exploration. Should mathematics be in the center of the inquiry? For me, exploring issues in mathematics education is my responsibility as a researcher and teacher educator. However, if mathematics were the only focus of the study, many important aspects of GEMS would be neglected. The data analysis is personal, dynamic, and iterated, requiring change and rechange, often including further reading.

Transferring Field Texts to Research Texts

Clandinin and Connelly (2000) asserted that the process of moving from field texts to research texts is far more complex than telling and retelling stories.

> A narrative inquirer spends many hours reading and rereading fields texts in order to construct a chronicled or summarized account of what is contained within matters such as character, place, scene, plot, tension, end point, narrator, context, and tone; these matters become increasingly complex as an inquirer pursues this relentless rereading. (p. 131)

Dewey (1938) encouraged researchers to be cautious in the data selection, because not all experiences are educative. An important process in transferring field texts to research texts is to read and reread field texts and selected educative data (Dewey, 1938). In this study, identifying educative experiences was not straightforward but was full of ambiguity. Some past experiences seemingly did not influence the current identity. For instance, Laura told her mother's story of being a single mother, which, at first, did not seem relevant to her experiences with GEMS. However, when I further read the field texts, I found that Laura's career selection, her identity as a woman, and the way she talked with parents about GEMS were, in part, responses to her mother's story. I recategorized the story as an educative experience, and I tell the story in the next chapter.

Transferring field texts to research texts was a messy process. I read and reread the field texts and listened and relistened to the audio recordings. I continued writing the reflections and memos, adding more complexity presented in the data. I categorized and recategorized the data and changed and rechanged the themes and plots. During the process, I revisited the boundary crossing theory and feminist theory to seek theoretical supports for deep understanding of the data.

Listening for Different Voices

In narrative inquiry analysis, a key consideration is the multiplicity of voices of both participant and researcher (Clandinin & Connelly, 2000). Lieblich et al. (1998) noted that a narrative researcher needs to look for three voices: the narrator's voice, the voice of the researcher's theoretical perspective, and self-awareness of the decision-making process of drawing conclusions from the data. Recording field texts helped me to explore my theoretical perspective and track the decision making from the data.

I also followed Brown and Gilligan's (1991) responsive listening for different voices in the data analysis process. From psychological and feminist perspectives, Brown and Gilligan suggested that the researcher listen to a participant's story four different times to capture different voices. The first time, the researcher as the listener attends to the story itself to understand it. The second time through the narrative, the researcher listens for "self," the voice of the "I" who speaks in the story. In the process of attending to self, the researcher becomes engaged or involved with the speaker. Listening the third time involves caring, listening, and responding. The fourth way of listening concerns equality, reciprocity, and fairness among people, including participants and researchers. I practiced the four-way listening method for Laura's first and second interviews. For each of the four suggested ways, I listened multiple times to seek better understanding of the voices. I heard Laura's voice, my resonation with her voice, my responses to her voice, and equity and social justice issues. The third interview was based on our three years collaboration and knowing well of each other. I still used the four-way listening method and heard both Laura's voice and my own voice in the first time. As I continued listening for the second time, I was able to hear the interaction between Laura and me regarding the issues we both were concerned with, such as learning and women's participation in mathematics/STEM.

Narrative Analysis and Analysis of Narrative

Polkinghorne (1995) identified two analytical approaches: *analysis of narrative and narrative analysis*. The analysis of narrative method employs paradigmatic reasoning to collect stories as data and analyze them in descriptions of themes that hold across the stories or in taxonomies of types of stories or characters. In narrative analysis, the data are events and happenings, which are formed into a coherent story. The narrative analysis method uses narrative reasoning to collect descriptions of events and synthesize or configure them into a plot in a story or stories. In contrast, in an analysis of narrative, the data are in the form of stories and are moved into common themes.

I employed both approaches to analyze data. First, I utilized a narrative analysis approach to generate three holistic plots: narratives of becoming female educators, boundary crossing collaboration during GEMS, and conceptualizing mathematics across multiple contexts. The three plots are presented in Chapter 4, Chapter 5, and Chapter 6, respectively. An analysis of narrative approach was used in each plot to examine Laura's and my stories in different life stages and across multiple contexts. For instance, in the plot of becoming informal educators, themes are identified about the topics of becoming women, teachers, and educators. In the plot of GEMS, themes that are identified include GEMS as an informal learning environment, GEMS leaders as informal educators, and GEMS as a space that empowers girls. In the plot of mathematics, themes that are identified consist of conceptualization of mathematics, mathematics in GEMS, and evolving mathematics in the collaboration.

Using analysis of narrative, I focused on the explicit content, that is, what happened, why, and who participated in the event. Also, I considered how the stories are told: what emotions the individual displayed, or what particular words were used by the narrator, which displays deeper layers of the narrator's identity (Lieblich et al., 1998). The themes are elaborated within our own social relationships and contexts, including personal and professional experiences, leading us to the current positions in the study. The combination of analysis of narrative and narrative analysis approaches ensures analysis beyond introspection and the collection of stories to construct narratives from the evidence that addresses social concerns of women and mathematics/STEM.

Throughout the data analysis, boundary crossing theory and feminist theory were used as an analytic lens and theoretical lens to support the validity. After writing each chapter, I asked

Laura to do a member check. Laura reviewed content, checked quotes for accuracy, and edited grammatical issues she noticed, which increased reliability of the analysis.

Potential Limitations of Narrative Inquiry

In narrative research, the researcher and participant both learn and change throughout the encounter. In the research process, they negotiate the meaning of the story and add a validation check to the analysis. The researcher's story is interwoven with the participant's story. In this way, the researcher gains insight into their own life (Creswell, 2013). However, the intimate interaction might be considered a weakness of narrative inquiry. In this section, I discuss both points of view and potential challenges of using narrative inquiry.

Crossing Cultures and Languages

Given the characteristics of narrative research, it can be challenging when researchers and participants come from different cultures (Creswell, 2013). I came to this study as an Asian woman who is an outsider within the Western culture. In particular, language, as a form of culture, conveys cultural information that leads the researcher and participant to exchange perspectives. English, as my second language, is a barrier for me in thoroughly understanding what Laura related to me. In narrative research, individual stories are situated within personal, cultural, and historical contexts (Creswell, 2013). Thus, cultures and languages could be barriers for a researcher like me, who crosses into different cultures and languages from my own to access true stories.

To address this limitation and have a clear understanding of the context of Laura's life, I collected extensive information about the contexts. I explored literature on girls' and women's experiences in mathematics/STEM over the past 50 years in the United States and beyond, including their experiences in K-12 education, higher education, and professional situations. In addition, I learned about feminist movements and feminist scholarship by taking feminist theory courses and communicating with feminist researchers. I also talked with Americans from Laura's generation to learn the context of school and society. I took several integrated STEM courses and collaborated with people from other STEM disciplines to explore the relations between mathematics and STEM. The knowledge shaped my listening and questioning during the inquiry of Laura's experiences and allowed me to see what was salient.

Ethics and Moral Dilemmas

Ethics plays a central role throughout and beyond the narrative inquiry research process. The trustworthiness and openness between researcher and participant can ensure a researcher's access to personally significant and meaningful memories and experiences. As Laura and I entered into an inquiry relationship, we began the ongoing ethical negotiation process and faced moments of ethical conflict. I endeavored to build a genuine, empathic, and respectful relationship with Laura (Josselson, 2007). This was a challenge for both of us to balance an equitable relationship. Laura often asked, "What can I do for your dissertation?" and "What do you need from me?" To avoid a receiver-provider relationship, I worked toward building a reciprocal and sustainable relationship within the study.

Another moral dilemma concerned whose stories should be told. Pinnegar and Daynes (2007) raised a set of essential questions: Who owns a story? Who can tell it? Who can change it? Whose version of a story is convincing? What happens when narratives compete? What role do stories play among us? As Pinnegar and Daynes emphasized, these questions are philosophical but, even more, methodological. The researcher collects stories from the participant, the text belongs to the researcher, and they interpret in a process called *re-storying*. In other words, the researcher takes full interpretive authority for understanding the stories and narratives (Chase, 2018). Every narrative contains multiple facets of truth, and certain facets are appropriate to some research contexts, while others are not. Reissman (2008) noted that each facet has a potential dark side. In fact, revealing the truth might "undermine ethical relations with participant communities" (p. 230).

Clandinin and Connelly (2000) explained that ownership issues blur into ethics concerns and negotiated relationships in the field. They clarified that *ownership* might not be as accurate as *responsibilities* to those in the researcher's circle. The researcher takes a risk by writing stories from a theoretical position, approaching the participants who are not coming from this position, and negotiating a single shared truth (Chase, 2018). Tensions might spring from the fact that the participant's stories are used as materials to achieve the researcher's goals. Although the researcher can explain the purpose and reveal their choices clearly and sensitively, it might not be aligned with the participant's self-perceptions. In this sense, the researcher might risk hurting the participant's feelings, surprising them, or influencing them for better or worse (Chase, 2018). I maintained my ethical obligations and conducted member checks with Laura throughout the inquiry to ensure that she confirmed my interpretations. Sometimes she pointed out conflicts of interpretation; she was straightforward and told me, "I did not mean this."

Chapter Summary

In this study, I sought to unpack women's experiences and marginalized perspectives from mainstream research. A caring and intimate relationship allowed me to capture nuanced information. I aimed to explore participants' lived experiences, understand the interaction between Laura and me, and value co-constructed knowledge. In this chapter, I articulated narrative inquiry as the methodology in this study. I began by defining narrative inquiry, drawing from multiple definitions, and described the process of becoming a narrative inquirer. I also reviewed relevant literature using narrative inquiry methodology in mathematics/STEM education and teacher education. Then, I explained the research methods used, including processes for data collection and data analysis. In the next three chapters, I share findings from analysis and unfold the three plots of the narrative inquiry: (1) narratives of becoming informal educators, (2) boundary-crossing collaboration in the midst of GEMS, and (3) conceptualizing mathematics across multiple contexts. Chapter 4, "Looking Backward and Looking Forward: Becoming Female Educators," presents Laura's and my narratives of becoming women, teachers, and educators, which provides opportunities for us to reconstruct our past experiences and reflect on how personal histories shape our personal and professional identities. In Chapter 5, "In the Midst of GEMS," I tell stories of GEMS as a learning space in a historical, social, and political context and explore the stories in GEMS from leaders' perspectives. Specifically, I explore GEMS as a space that empowers girls and women in STEM. In Chapter 6, "Mathematics across Boundaries," I explore Laura's and my mathematics experiences and evolving views of mathematics to illuminate our conceptualizations of mathematics. I describe mathematics in GEMS in the past, present, and future to understand the role of mathematics in GEMS. In particular, utilizing a boundary crossing lens, I describe emerging mathematics in our collaboration.

CHAPTER 4. LOOKING BACKWARD AND LOOKING FORWARD: BECOMING FEMALE EDUCATORS

In this chapter, I first explore Laura's family stories and juxtapose my personal stories to understand experiences of being and becoming women. For Laura's stories, I draw from interviews, historical fictions, *The All-Wise Being* (2009) and *Eel River Rising* (2014), Laura wrote, her own narratives, and literature. For my stories, I draw from personal journals, reflections, and family stories told by my parents. All of our stories are situated in particular historical, social, and political contexts. To understand each other's stories, we cross both cultural and historical boundaries. Resonations and discordances are normal in understanding each other's stories; we explain and re-explain to each other and gain deeper understanding about each other's narratives. Then, I unpack our experiences of becoming teachers to understand teacher identity in our own respective cultural and social contexts. Finally, I describe turning points in our professional lives that led us to become educators. These experiences have shaped and continue shaping our personal practical knowledge as educators (Clandinin & Connelly, 2000). Looking back provides opportunities for us to reconstruct our past experiences and reflect on how personal experiences and family histories shape our identities as women, teachers, and educators. Looking forward allows us to forge new identities in the collaboration that further informs our teacher knowledge and teaching practices.

Becoming Women

Being a woman is a social construct. Its embodiments are evolving together with social and cultural change. A woman is not a fixed biological category; rather, being a woman is "a becoming, a condition actively under construction" (Connell & Pearse, 2009, p. 6). The lives of women often include roles as daughters and mothers that expand the socialization of women's experiences. Laura often tells people why she started GEMS: "I started GEMS for my daughter, Janet. She said, 'Math is hard.'" To some extent, founding GEMS for her daughter was a kind of mothering practice for Laura. In this section, I first present Laura's family histories from her perspective to describe her experiences of becoming a woman and her views of women, in particular, the equality of women. Then, I share my own personal and professional narrative to unpack the impacts of contexts that shape my experiences of becoming a woman.

Laura's Narratives

Family and educational backgrounds nurture who we are and whom we are becoming (He, 2002). Narrative inquiry is a mode of inquiry that would continue the flow of recovering and reclaiming stories of family and community. Telling stories of experiences from family allows us to discover the development of fluid and dynamic identities and to explore how the experiences become embedded in shaping and informing our teacher knowledge. It is my desire that readers too, can become familiar with Laura whom I engaged in my inquiry, by being situated in her context to understand her development of identities and teacher knowledge.

I started with Laura's narratives from her mother who had a unique influence on Laura's education, domesticity, and career direction. Mothers are believed to be more significant models for girls, mothers' behaviors, including what they do and say, are important to know because these actions communicate messages to their daughters about personal and social values (e.g., Looker & Magee, 2000). Then, I briefly tell Laura's educational experiences that lead her career direction of becoming a teacher and educator.

I tell Laura's stories of marriages in which Laura reconstructed her experiences of being a wife and a mother which provided her an opportunity to reflect on her evolving identities in family and society. I also describe Laura's experiences of mothering through which Laura communicates her values of women and education with her daughters. Finally, I describe an angry woman image portrayed by Laura in her historical fictions that mirrors her awareness of the oppression of woman across contexts.

Laura's narrative experiences I share below are educative that informed her identity as an informal educator and her teacher knowledge (Dewey, 1938). On the one hand, this is my intention to provide rich contextual information about Laura to readers and myself knowing her as a real person. On the other hand, these experiences have shaped her personal practical knowledge that informed her to establish a learning environment (i.e., GEMS) and choose curriculum for GEMS (Connelly & Clandinin, 1988).

Her Mother

During the 1960s and 1970s, Laura's mother reared five children as a single mother. Laura is the second child. She has an older sister, two younger brothers, and one younger sister. When

Laura was 8, her parents divorced; her father left home and the children remained with her mother. Laura's mother worked for the State of Indiana in the unemployment office in downtown Indianapolis for 35 years. Her work was to review unemployment claims. She graduated from high school in 1941, just before the beginning of World War II. She did not have the opportunity to attend college. Because she did not have a college degree, she trained people who were later promoted, which happened repeatedly. She, herself, was never promoted.

Driven by fear, Laura knew her mother had high expectations for her and other siblings. Laura's mother was afraid that her children would be unable to support themselves. She especially never wanted the girls to be like her, abandoned with children. She told the three girls in the family to be nurses or teachers because they could always have a job. To urge the children to do well, she made them feel guilty if they did not perform well in school. When Laura was nine years old, she received scores in the 99th percentile from a school test report. She was excited to show her achievement test scores to her mom. Her mom looked at the grade sheet carefully and finally said, "Laura, you always miss a point!" As a little girl, Laura felt guilty that she did not get 100%. Because of her mom's strict expectations, Laura worked hard for good grades and obeyed rules at school. She said:

> I really felt like mom wanted me to do well, so I'm gonna do well. I was not a person who took risks in school. Seriously, I wasn't gonna fool around and write a paper on something weird because I wanted to get a good grade. I'm serious. I was very quiet in school, in fact. (Laura, Interview, August 2019)

When Laura was a high school sophomore, her older sister was a junior. Laura's uncle, an elementary school principal, brought information about college scholarships to Laura's mother. Laura recalled:

> My uncle said to my mom, 'there is money for college, you need to head these kids toward college. . .. I think he saw that the five little kids with no money but are pretty smart and my mom really wasn't thinking ahead because she didn't have college experience. . .. It changed everything, because my mom was thinking like her, we would get out of high school and get jobs. We weren't planning for college at all. (Laura, Interview, August 2019)

Laura's mother immediately took action and filled in all the financial aid forms to apply for scholarships for her children. Eventually, the mother's efforts paid off; all five children were admitted to college with full scholarships and completed advanced degrees.

Laura said:

> I think back, that is my mom's intelligence. She used to laugh and said, "The only thing I ever gave you was financial need." Yes, you did, Mom! I think my mom is a strong woman role model [for me], even though she wasn't college educated, she wasn't in science, it's not like somebody was a nuclear physicist. But she is a real solid person and that was one of the biggest helps. (Laura, Interview, August 2019)

Until Laura grew up and had her own children, she did not recognize how strong her mother was or understand what it would be like to be a single mom raising five children with one income. Laura's mother provided a role model for her life, leading her to become a strong-minded woman.

Laura's mother never really talked about her education. Laura felt that she did not know enough about her mother. She expressed:

> I would like to write a book about my mom's time—not about my mom, but about a girl who, because of World War II, everything changed, and about what happened to women in the '40s. It was awful because they got to do really cool things during the war and then when the war ended, that was all taken away from them and the men got the jobs back. I could see that happening to my mom, but we don't really know anything about my mom. She never talked about anything, ever. She got married really late, at 29, which was very late and had us kids right away. I just think she was desperate. I'd like to write. It wouldn't be her story but about her time. (Laura, Interview, August 2019)

Writing this book is a way to understand her mother in her context. Perhaps because it is so emotional to think about, Laura finds it hard to get started.

Education

Laura was a high academic achiever. In the fourth grade, she scored at a high level in a statewide test, and she was invited to the psychologist's office to take an IQ test and be interviewed. After she went through these processes, she was informed that she qualified for a gifted class, which consisted of 20 students from schools all around the city. The students in the gifted class had the reputation of being smart kids and were separated from other students in the fifth grade to the eighth grade. They were provided more advanced curricula than other students in the same grade level.

Because Laura was growing up with siblings who were close in age, competition was hard to avoid. Laura completed her bachelor's degree in three years at Purdue University. She achieved this as a result of competition:

> I was competing with my sister, who is a year older than me. She started a year before me, and I was determined. "I'll just beat that, so we'll graduate the same

> year." So, it sounds really exciting, but the motivation was not good. (Laura, Interview, August 2019)

Laura further described her competition with her older sister:

> I think I competed with her; she didn't really compete with me because she was older. This is really bad. When we grew up and got away from each other, we realized that people saw Melanie [Laura's older sister] was the pretty one, Laura was the smart one, and Rhonda [Laura's younger sister] was the nicest one. How terrible is that, first of all, we're all smart, we're all very well-educated, we're all nice. It is awful! (Laura, Interview, August 2019)

In college, Laura worked hard to maintain her scholarship. She felt it was unfair that her brother spent less effort but still got scholarships. She told:

> My brother Martin was right under me, and he went to Indiana University, and I went to Purdue. We all had scholarships. I was determined that I would earn that scholarship. I mean, I would do my very best—even though I fooled around in science and got a C in one—because I don't want to lose the scholarship. He skims by with Cs and never loses his scholarship. I was up here determined never to get any bad grades, but you're skimming through with Cs and they give him the scholarship. It was so annoying. (Laura, Interview, August 2019)

After graduation from college, Laura then got a scholarship from Syracuse University for her Master's degree program in special education. She told:

> I got a full assistantship there to work under [a professor who is famous in the special education field], and I had never heard of him. I didn't even realize what an honor it was to work with him. I think it was a real waste for me, but I learned a lot, and so then I—then I got out of school and became a special ed teacher. I would not have been a very good special ed teacher without going to Syracuse and getting a master's in that. . .. After I got out of Syracuse, I realized, boy, I wasted that opportunity—I think in the back of my head was always "I've got to raise my kids, so they don't turn down opportunities." (Laura, Interview, August 2019)

Since 1974 when Laura graduated from her master's program, she worked as a special education preschool teacher until 2001, when she received the National Board Certification and changed her career to become a technology specialist. She explained:

> I went into preschool special ed, then I realized I am a person who likes to see a lot of progress. You did see progress with your preschoolers. Our goal always was to get them in and work on the speech or the fine motor and get them out. Then you brought them up to a normal level of development and I think that probably kind of characterized the rest of my life. And I think that's why I was kind of glad to move out of special ed and get into technology. It [technology] also changes all the time. You're always learning something new. I think that's important to me. (Laura, Interview, August 2019)

Laura is a lifelong learner; she enjoys learning and supporting other people in their learning. She reaps benefits from education and recognizes the power of education. Through GEMS, Laura supports children, in particular girls, to become who they want to be. To see students making progress gives Laura joy and satisfaction.

Marriages

Laura's first marriage ended after 17 years. She did not regret the divorce, saying, "I married too young—we grew into two different people. I have my wonderful children, so I have no complaints there." Laura was expected to be a traditional woman who prioritized children and husband rather than self and career. Laura recalled that the minister who facilitated their premarital counseling said to her, "When you get married and have children, of course, you'll stay home with them." She described her reaction. "I'm like, no! I was deeply angry." Laura shared another story:

> You know what my first husband said? We both had full-time jobs, and the kids were little then, and I'm working for the schools, and one time one of them was sick. I had a full-time job. We had times when we had to be at work. Other times we could reschedule things. My daughter was sick, and he actually said to her, "Your mom's going to stay home with you, because I have a real job." (Laura, Interview, August 2019)

A few years later after her first marriage and she had changed her career to a technology specialist, Laura said, "He [Laura's ex-husband] was a numbers guy. We had a really old computer. But he never let me use the computer. I didn't know how cool it was going to be." Later, she found "it [technology] is like a part of my life. I am really good at it. I look back and think that was a better fit for me. But that wasn't available when I was in college."

Her second marriage provided Laura with many opportunities for self-development. When Laura decided to work toward the National Board Certification as a teacher (more details about this event later in the chapter), her new husband encouraged her and supported her to achieve her goal. Laura recalled:

> Doing National Board Certification was a risk because it's so much work and it's expensive. He would always say, "Go for it," from the very first thing with that National Board. He always pushes me a little, like taking that job with the National Board. He was like, "Go for it, you can do this." Now I don't even have to discuss with him. I know what he's going to say. When I did National Board Certification, we had a big house and I only had one child at home then and he cleared out part of an upstairs room, brought in a huge table and a chair, and said, "This is where

> you're gonna do it." I have to say, that started the ball rolling. (Laura, Interview, August 2019)

After she got the certification, the National Board invited her to work at their national office in Arlington. She was hesitant to leave her comfort zone and take risks in an unknown area. A simple "go for it" from her husband gave Laura great encouragement, which led to a considerable change in her career trajectory. Laura always uses "wonderful" to refer to her second husband; in her second marriage, Laura became a more independent woman. She started the GEMS club and changed the career in which she had been working for 26 years, doing more challenging work, such as learning technology that she had never experienced before.

Mothering

Laura has two daughters from her first marriage, Colleen and Janet, who were born six years apart. Being a mother is a huge part of Laura's life. Laura described that she was able to do the National Board Certification because her daughters did not need her attention at that time. She explained, "by the time I did that National Board thing, Janet was in high school. See, so things were very smooth, and Colleen was out of the house at college. Otherwise, I would have had to concentrate on my children." Like many parents, Laura knows each of her daughter's strengths and weaknesses. In Laura's eyes, the two daughters are very different; Colleen likes to challenge herself, while Janet always needs a little push. Laura told:

> I remember *The Oregon Trail* computer game. Colleen would have been eight or nine. She'd take up the disk, put it in the computer. She would figure out how to play it, and I used to say—before I knew better, I would say she approaches it like a boy, but it is true. That's what boys do. Girls read the manual. She was up there playing *Oregon Trail* and having all this fun. She would always challenge herself: "Give me a new game, I'll figure that one out." Janet has no interest. Janet was just, "Life is good; it's going to be okay." Janet never worked very hard and got good grades. She was like a B+ student but could have been so much better. You've got to push her. (Laura, Interview, August 2019)

Laura laughed, "I always say I could take those two kids, smash them together, and then Colleen would be a little calmer, and Janet would have a little more energy."

Being a teacher and educator herself, Laura particularly cares about her daughters' education. As with many issues, Laura supported Colleen and Janet to complete their college degrees. Laura told Colleen's story of pursuing her college degree:

> Colleen got ADD from her dad, anxiety from me, and was very emotional. And then we got divorced, and it just knocked her crazy, and school was so hard for her. And I tried so hard to get services for her, but at that time, you had to fail before you could get services. So, we had her privately tested. By God, yeah, she was learning disabled, but by then she felt so badly about herself that she pretty much gave up. The only good thing is she didn't give up enough to really blow it. She got through college. Colleen went to a good state school in Virginia, and she finally wised up and got some help. She told them, "Yes, I am learning disabled. Yes, I need help." So that was good. They worked with her, and they got her through school. She got her degree, and no one could ever take that away from her. (Laura, Interview, August 2019)

To encourage Janet in math and science, Laura started the GEMS program in her elementary school, which I will describe in more detail in the next chapter. Although there were different issues from Colleen, Janet's college education was also a challenging journey. Laura explained:

> Janet was really good at math. She would take statistics as an elective. She got admitted here at Purdue in Engineering. At that time, it [tuition]was $22,000. I was a teacher and [Laura's second husband] was an unemployed minister at that point. I asked for financial aid. We filled out the forms, and they're like—no. Because we lived in Virginia, we made a lot of money in the eyes of the people here. But we couldn't afford it. So, she went off to the other university in Virginia. Janet was disappointed, and she hated it. She flunked out and she came back and went to community college and immediately got straight As. She was saying, "I should have done this first." Janet couldn't handle the lectures—big lectures of math and chemistry. She did extremely well in the community college, and then she transferred right to a local large university and did very well. (Laura, Interview, August 2019)

When it was time to choose a major, Laura encouraged Janet to follow her own heart. She laughed:

> But what was funny—when she had to declare a major, she didn't know what she wanted to do, because it wasn't engineering anymore. She saw event management and hospitality. She's like, "Oh, that sounds good." This is what she gets her degree in. She has got this nice degree and did very well and she hates it. This is just so convoluted. She started her [career] as a receptionist. Then she was a contractor for a computer forensics company. Until she left to stay home with the kids, she was a computer forensics analyst. Then she's decided to try and become a food and nutrition coach and maybe kind of just have clients. (Laura, Interview, August 2019)

Last summer, Laura was excited to tell me that Janet would pursue a full-time master's program in nutrition. Laura said:

> I am so proud of her. She got those certificates from these online places in nutrition to help people, then she started to realize that without a degree, she really has to be careful. Who are you to recommend what I do when you don't really have a degree in it? . . .. She applied to graduate school and then she got in and I never thought

> twice—of course, she'd get in. But she had forgotten what good grades she had, which is another exciting thing. That's all her idea. (Laura, Interview, June 2021)

Since Janet returned to school, Laura is back to a mothering position and helps Janet out by visiting her often and taking care of the children.

Angry Women

Laura is also a writer. In 2009, she wrote her first historical fiction book based on her family stories, *The All-Wise Being*. In Laura's family, her great-grandfather, Ethan, was a lawyer and an Indiana State Representative for a year. Based on Ethan's diary, which he wrote in every year on his birthday, and family stories, Laura looked up newspapers, journals, and court documents and portrayed Ethan's whole life, starting from 1863 when Ethan was 12 until he took his own life in 1913. When she finished the book, Laura realized the only thing she knew about Ethan's wife, Flora, was that she was a teacher. Other than that, she knew nothing about Flora. Laura said, "Flora became kind of radical. She raised the kid and Ethan was out. She doesn't have anything to do because she can't work after marriage, of course." Laura said, "Then I decided to write her story, and she was an angry woman."

In 2014, Laura's book *Eel River Rising,* about Flora's life, was published. Flora dreamed of becoming a doctor, even though she knew her path would be full of obstacles. Eventually, Flora decided to become a teacher. Writing Flora's story, Laura incorporated her own struggles and resistances:

> I joke about this—I call this [*Eel River Rising*] my feminist manifesto, but I think it was the whole story of—here is an intelligent woman who has had no opportunities. You know what I mean—she wanted to be a doctor, you see, but she could not go because her dad died. That was made up. But she became a teacher like any other smart woman back then—that was the only thing they could do, which I think is my story too. I mean, I guess maybe there is a prototype. (Laura, Interview, October 2019)

I asked Laura when she started to be aware of women or gender inequality issues. She said,

> I don't think there was anything my mom did. It's not like she treated the boys better than the girls. But I think I saw it all the time that the boys were more valued . . .. I was an angry young woman back in high school. I mean, I think I was. . .. Those were the times when women were beginning to stand up for themselves. I think, I just, it was like, I am here. (Laura, Interview, August 2019)

In the books about Ethan and Flora, Laura focused on the stories in the time of the late 19th and early 20th centuries. She described historical events (e.g., the Civil War) and social context (e.g., women's suffrage) that influenced individuals' lives. In particular, from a woman's perspective, she depicted how women were treated and women's resistance of injustice in family and society during the first wave of the feminist movement. In *The All-Wise Being*, Laura reported from Ethan's diary, "Flora amuses me—she is for suffrage but knows my feelings—I cannot approve. Women do not have the intellect, reason, or education to vote wisely. They should listen and heed male decisions" (Jones, 2009).

In *Eel River Rising*, Laura wrote that Flora attended the women's suffrage parade in 1913 at the time of the inauguration for Woodrow Wilson. She came back from the parade very distressed and disappointed because of how she was treated. Laura explained,

> Nothing bad but, I mean, just the thought that the anger and hate which really happened directed toward all these women, there was so much anger and hate. And that was seven years before they finally passed the amendment [Women's Right to Vote]. (Laura, Interview, October 2019)

Laura's anger about the status of women emerged during the second wave of the feminist movement. Influenced by the civil rights movement, women's movements began focusing on women's rights as civil rights, encompassing workforce discrimination, pay inequality, reproductive rights, domestic violence, and women's studies (Baxandall & Gordon, 2002). In second-wave feminist movements, feminist thoughts emerged concerning "the equality of, and differences between, women and men" (Evans, 1995, p. 2). Laura described herself at the time:

> Look, in the whole world, you know that women were just not [treated equally]. Women didn't have and still don't have the rights that men have. I mean, that was the beginning of the ERA [Equal Rights Amendment], and it still wasn't passed. I was like 16, so, as I said, I've been an angry woman for a lot of time. And I'm not angry personally. You know what I mean—it's not personally turned [on] anybody. (Laura, Interview, August 2019)

In her daughters' eyes, Laura was an angry mother. She did not allow her daughters to order food from a specific restaurant because the owner was against women's abortion rights. She said, "I'm an angry mom—that's right! There is no equity here." She laughed, "It's funny my kids used to say, 'You're just a "femi."' When her daughters grew up, they began to understand their mother. "As they got older, they realized, 'Oh, mama, I see what you're saying.'" Laura told a story about her older daughter's experience with injustice:

> She [Colleen] actually sued one of the companies she worked for in sales because the men were getting bigger bonuses than she was. But she had a better sales record. So she sued them. Of course, she can't work there anymore. (Laura, Interview, August 2019)

As a parent of two girls, Laura tried to provide her daughters with different options for playing: "They have a house full of naked Barbies, but we had blocks, we had trucks, they had Legos, and they had the video games." As both a parent and an educator, Laura was also angry about "the low number of girls in these [advanced math and science] classes, which means the low number of girls who have the opportunities to get really good jobs in the future."

Lili's Family Stories

Women in My Family

My grandmother was born at the same time as Laura's mother. She passed away when I was one year old. I heard many stories about her from my mother. At the age of eight, my grandmother started foot binding, which was prevalent in Chinese society at the time. Her toes were snapped and folded; the feet were tightly bound by long, narrow fabric strips, so they did not grow anymore. As her body continued developing, the three-inch feet could not support the weight of her whole body. Thus, she took each step carefully, tentatively, and every step was painful. She could not walk too far and, most of the time, she stayed at home doing housework. At the age of 28, a miscarriage caused her left leg to be paralyzed. After a year of recovery, she relied on her right leg for mobility for the rest of her life.

Foot binding was abolished before my mother was born, but emotional and moral constraints were still prevalent. My grandmother educated my mother to obey her father and husband as a mark of a woman's virtue. During the cultural revolution, my grandfather was accused of being counter-revolutionary because he worked for the previous government; he suffered public humiliation. His two children, my mother and my uncle, were deprived of opportunities to access education and apprenticeship training. My mother showed her bravery and wisdom, faced the Red Guards, and protected the family. My grandfather often lamented that my mother was a daughter, not a son, so her talent was a waste.

My mother has a natural passion for learning. She received an old dictionary from a village teacher. Relying on the dictionary, she read all the newspapers and books she could find. During the cultural revolution, not many books were available in the village. She even borrowed medical books from a village *barefoot doctor* (i.e., a village doctor at that time who did not have much professional training). Reading helped her know more about the world, and she began to contemplate the meaning of life. She was not satisfied with farm life and desired a change. However, she did not have any degrees or skills, making it impossible to find a decent job.

Her marriage was arranged at the age of 28 by a relative, which was considered very late for the local area. My father has a high school degree and was a teacher in his village school which was 50 miles aways from my mother's. When we lived in the village, my father taught school and did some farm work before and after school. My mother worked at home and on the farm, feeding chickens, pigs, cows, and sheep. She was willing to do more housework because she admired my father's profession and believed he was a knowledgeable person and deserved more respect and less laborious work. Unfortunately, my father later became addicted to gambling; he often disappeared for several weeks without informing us and left all the work to my mother. My mother constantly struggled with her marriage. However, obedience was already infused in her; she thought about divorce but never followed through with it. Even today, she believes that tolerance in her marriage was the right decision. She regards staying in the marriage as a sacrifice for her children because she could not have reared two children by herself.

Like Laura's mother, my mother pushed me hard in education. She expects me to have skills, be independent, have my own career, and not rely on a husband. On the other hand, my mother expects me to be a quiet and silent girl and be a traditional woman who contributes to the family. Like my grandmother's expectations for her, she wants me to obey the norms of selfless devotion to family and children. In particular, she believes that a woman should be unconditionally subservient to men in the family, such as husbands and brothers. My mother still struggles between ingrained ideas and awareness of spiritual independence. Father, brother, husband, and son are the center of her life, and she does not view her own needs as important. When I once told her that women are not born to owe something to men, she commented that I was a selfish woman.

When I decided to pursue a doctoral degree in the United States, many people, including my friends, family, and colleagues, questioned me: "As a woman in your mid-30s, what do you want?" That was the exact reason that drove me to the decision, challenging the societal norms that expect a woman to focus on family, children, stable life, and not her own development. I learned these social norms from observing my mother's and other women's lives. Sacrifice is a key criterion to evaluate whether a woman is good or not. If a woman maximizes her sacrifice for the family, she will be considered a good wife, good mother, or good daughter.

What a Woman You Are!

Being a mathematics teacher in a school with excellent academic performance, I was recognized as a good teacher because I was patient, diligent, and cared for my students. Even though I took a leadership role as Grade Department Chair, people appreciated my dispositional character more than my intellectual contribution as I was succeeding in the position. Many people, including my friends, family, and colleagues, questioned: "As a woman in your middle 30s, who does not focus on family, husband career, children's education, which is selfish and unwisely". In a diary I wrote:

> I was on an airplane flying 30,000 feet above the Pacific Ocean traveling from China to the United States. My daughter feebly lay on me. She had a high fever, and I gave her Tylenol before boarding the plane. Apparently, her temperature was going up again. I took her in my arms and asked the flight attendant for a cup of warm water for her. Looking out the window, I saw the plane was flying through the clouds. This was not the longest journey of my life. When I was in high school, I stood for 20 hours on a train to visit my parents, who sought work opportunities in a city far away from our hometown. But this was the first time I had gone somewhere unknown. In a trance, I could not tell where I was or where I was going.
>
> Memories sprung up. Walking out of the school Human Resources office, I had indescribable feelings. Knowing I came to resign the job, the HR representative asked, "Why did you decide to study abroad at this age?" A short conversation made me feel that I had made the riskiest and stupidest decision. I went back to my classroom; my students were excited to hear that I would go to the United States for a doctoral degree: "Ms. Zhou, will you still teach math in the United States?" "Please don't forget us." "Remember to take Chinese snacks with you when you go to the United States." "Will you come back when you are done?"
>
> In the school dining hall, I was having lunch with a close friend and colleague. We had worked together for eight years and became mothers around the same time. "Are you sure you want to go to the United States and leave your husband alone here? What a woman you are!" What I heard was a critique of me as a selfish woman who did not prioritize family. I told her that it was a family decision rather than my own decision. She was not convinced and shook her head. "I admire you, but I would never do what you do."
>
> A week before we left, my husband was busy with his work, but he came home earlier than usual. We did not talk much about leaving. He said, "It's not bad you

> are not around. I could concentrate on my work and with fewer distractions." We imagined the reunion in the winter and somehow were excited about the unknown world. In the airport, he said, "Come back any time if you feel it is difficult to handle." I replied, "Okay, but I will try not to do that." The flight attendant brought over the water. My recalling was interrupted. (Lili, Diary, October 2017)

Pursuing a PhD degree is not encouraged but is perceived as an unusual decision that challenges the social expectations of a mother and a wife. As I reread the diary, I still can feel the moments that people questioned my decision.

Becoming Teachers: Across Cultural and Social Contexts

Teaching is a typical career for women across many cultures. Although Laura and I live in different contexts, both of us chose to be teachers, which is of the foundation for our ongoing collaboration. Laura chose to be a teacher in the 1970s in the U.S context, which was not her intention, while I chose to be a teacher in 2000 in China, which was my decision. Exploring our stories of being teachers provides an opportunity for Laura and me to reflect on our past experiences to understand the cross-cultural experiences of our collaboration. More importantly, looking backward makes individuals' experiences go beyond the personal level and leads to an inquiry about women's access, opportunities, and identities in specific contexts, setting a stage for looking forward.

Laura: Becoming a Teacher Is to Be Real

Laura went to college in 1970. In the first year of college, she struggled with choosing her major. Laura's mother always wanted her to be a nurse or a teacher. "Being a teacher was a typically female job at that time," Laura explained. As a single mother with a high school diploma raising five children, Laura's mother knew how hard it was to support a family. Laura's mother set expectations for her daughters that being a nurse or teacher could always supply a job so they could support themselves. However, Laura's true love was not teaching. She had a passion for chemistry and wanted to be a pharmacist.

As a first-generation college girl from a lower-class family, Laura relied on authority figures to give her suggestions and direction. Early in her freshman year at college, she went to the Dean of Women's office. She talked to a staff member about changing her major to pharmacy.

The staff member told her, “Honey, if you want to raise a family, you can’t be a pharmacist.” Influenced by her mother, Laura feared she would be unable to raise a family. Although her true dream was to be a pharmacist, she listened to the authority’s voice and chose to become a teacher. However, deep in her heart, she felt that “the opportunity was turned down.” In an interview, she regretted, “I was young and stupid. I was 17, and I was like, ‘Okay, I’ll be a teacher.’ But it [teaching] wasn’t my passion.”

Laura struggled with her decision to choose the teaching profession despite the gender role stereotype that viewed teaching as a typical women’s job. She experienced a conflict between societal expectations and the self. She did not see herself as a good fit in the teaching profession. When she received a suggestion from an authority figure, she trusted the decision and embraced the societal and gender norms. She was clear that it was not her passion. Although later it turned out that Laura was a good teacher, she regrets losing the opportunity to pursue her dream.

Laura sighed, “I was born too early to become what I could have been.” Laura said this because two years later, in 1972, Title IX was passed. Title IX required that “no person in the United States shall, on the basis of sex, be excluded from participation in, be denied the benefits of, or be subjected to discrimination under any education program or activity receiving Federal financial assistance” (Hepler, 2013, p. 442). “It changed everything,” she said. Laura used her sister’s friend Ava as an example to illustrate how things changed so fast. Though she had Bs and Cs in high school, Ava was recruited in engineering right after Laura graduated from college. Thus, Laura believes that Title IX made a significant impact on women’s career decisions.

Title IX emerged and expanded during the second wave of feminism, which sought to eliminate gender-based discrimination in education. Title IX was a necessary mechanism that stimulated changes in promoting gender equality in educational fields (Rolison, 2003). Specifically, given the underrepresentation of women in STEM, Title IX was extended for combating gender inequities in STEM, including K-12 education and the academic and non-academic workforce (e.g., Walters & McNeely, 2010). In light of the policy environment, Laura started the GEMS club to provide a girls-only environment where beliefs in the equality of girls’ and boys’ capabilities and the differences in girls’ learning are emphasized.

In an interview, Laura said proudly, “Even though teaching was not my primary choice, it turns out I was good at it.” In an educational journal article, Laura stated:

> Good teaching is difficult to measure. It is almost impossible to define, let alone quantify. . .. Its [National Board for Professional Teaching Standards] mission is "to establish high and rigorous standards for what accomplished teachers should know and be able to do." That sounded like a good mission to me, and a good challenge. (Jones, 2001, p. 19)

Being a teacher is not just having a license or certification. Laura acknowledged:

> Being a teacher is so hard. That learning curve is so steep, and so many people don't make it. They don't make it over the hump into "I can just get better every year." They're—it's so hard to get to that point of being comfortable. (Laura, Interview, August 2019)

Lili: Being a Teacher is My Dream

I listened for the meaning and understanding of Laura's stories of becoming a teacher, and I began to reflect on mine. I was flooded with memories of my own stories of choosing to be a teacher. As I mentioned in the first chapter, choosing to be a teacher was a natural decision. Indeed, I did not have many options other than being a teacher. Being a teacher or doctor are two typical, decent jobs for girls. As a first-generation college student, I did not get much advice from my family and did not have a role model from family and my community other than teachers. Looking back, I never even thought about other career options that could be available to me. I was not interested in medicine and had a passion for teaching. Choosing to be a teacher became the best choice for me. Other than my father, my family was happy to see I chose to be a teacher. My father warned me that teaching is a huge commitment, that you need to take responsibility for others' futures. The warning did not dissuade me; instead, I committed to a lifelong career path in teaching.

Teaching is not a high-paying job. However, compared to my parents, who did not have a stable income, the monthly wage ensured I could make a living, and I even could support my family. After I graduated from college, I taught in a high school with a reputation for high college enrollment rates. I suggested that my brother transfer to my school from a vocational school to increase his chance of getting into a better university. In addition to the financial support, I believed that my being a teacher in the same school my brother attended could provide him a better opportunity to easily access educational support because he could reach out to my colleagues and me. This was also a way to pay back my parents, who had supported my education.

Growing up, I had become accustomed to being quiet and "invisible" among people. However, as a teacher, I had to make myself the center of attention in front of my students; I had to make on-the-spot decisions; I had to be prepared to answer questions from students. In other words, I developed agency in teaching to be a person who has authority, knowledge, and power. The energetic, excited, and inspiring image of a teacher was my self-image, which conflicted with a quiet and invisible image. As a teacher, I continuously negotiated the tensions between self-image and cultural expectations of women to create a new space for me to become the person I would like to be.

Specifically, the identity of the female mathematics teacher provided a space for me to express my understanding of mathematics, which was viewed as objective, abstract, and difficult, such that I recognized my inner source of strength as a woman in mathematics. Though I was not able to determine what to teach because I had to follow the school's curriculum schedule, I could decide *how* to teach. I integrated my understanding of mathematics into teaching by rearranging the content in the curriculum and adapting textbooks. In addition, I used multiple ways to motivate students in learning. I tried different approaches to make mathematics more engaging and accessible for students. Teaching is a mutual process; I teach my students and I also learn from my students and learn from teaching them.

Becoming Educators: Turning Points

Laura and I both changed our careers during our teaching lives. Here I use the term *turning points* (Drake, 2006) to indicate the significance of the changes. Getting National Board Certification was a turning point for Laura. After that, she left her comfort zone and started to learn technology and to support teachers in their use of technology. Studying abroad was a turning point for me. I left the school where I had been working for eight years and changed my role from a teacher back to a student. Exploring the turning points that led us to become educators helps us further understand each other's past experiences and develop reciprocal relations.

Laura: Getting National Board Certification

In 2000, Laura was a special education teacher in a public preschool program close to Washington, DC. People in the town read *The Washington Post*. Jay Mathews hosted an education column that frequently disseminated negative information about teachers. This information distressed Laura. She wanted to do something to counteract the unjust and inaccurate messages.

That was the first year that the National Board issued certificates for special education teachers. Laura decided to try to earn a certificate to prove that there were good teachers, and their voices should be heard. In her diary, Laura wrote:

> Why would anyone with two busy teenagers and a wonderful husband spend a year to get recognized as a nationally certified master teacher? I don't really know . . . I think the certification attracted me because I'm tired of the bad press teachers constantly get. I was educated in public schools, I send my children to public schools, and I teach in a public school system. I guess it was just time to stand up and declare my pride in my work and my workplace. (Jones, 2001, p. 19)

During the whole year, Laura took videos of her teaching, analyzed those videos, and wrote papers to meet the standards for certification. At the same time, she wrote a diary about her journey. All her hard work was rewarded. Laura recorded the moment that she found out the result:

> I make the morning rounds of students and come home around lunchtime to check the Web site again. It takes a while, but finally a list pops up. As I scroll down I see three friends from the support group. But I can't find my name. All that work! I start to cry. Then I realize—it's not in alphabetical order; it's listed by town of residence! And there I am under Herndon, Virginia: Laura Reasoner Jones. I keep crying. I am dazed. (Jones, 2001, p. 35)

When she received her certification, Laura contacted Jay Mathews and told him the story. Later her diary about being certified was published in *The Washington Post*.

Although Laura was not passionate about teaching in the beginning, she was an excellent teacher and loved it. Like women teachers in Munro's (1998) study, teaching was a space for Laura to define herself by reconciling the conflict between self and the societal image of teachers. At that time, Laura was middle-aged and could use reasoning and procedural knowledge to defend herself (Belenky et al., 1986). Laura paid attention to the external voices about what people thought about teachers at this life stage. As a member of the teacher community, she cared about what the image of a teacher was in people's minds. She was aware of using her knowledge against negative information. National Board Certification was a means to advocate for other teachers and herself. The conflict between her identity as a teacher and teacher image in public catalyzed her resistance.

It was a turning point in Laura's professional life; as she said, "It changed everything." Laura reflected, "Completing that process made me see myself differently and made me want to contribute in a different way" (Jones, 2005, p. 64). As a result of achieving the National Board Certification, Laura was invited to join the Digital Edge Project and work for the National Board as Teacher in Residence with ISTE and Apple Computer to develop an online teaching library of accomplished teachers integrating technology into their teaching. For two years, she had the opportunity to learn and use emerging technology. She learned how to make and edit videos and make websites.

At the National Board, Laura used a laptop for the first time. She couldn't even open it and sat in front of a group of other teachers for a long time to figure out how to use it. Laura called the two years she spent on this project an "adventure." She tried new things and gained new knowledge and skills, unlike working at school for a good grade or working for a certification. Laura explored new fields by working on projects. At the same time, she developed knowledge about technology and teaching. By participating in the project and learning, her identity transformed from a preschool teacher to an educator who focuses on supporting teachers and inspiring students.

After the Digital Edge Project, Laura went back to her preschool position. But she did not see herself fitting into this position anymore. She described her feeling at that time:

> It isn't that I was ignored; it was that what I had learned was ignored. I felt very frustrated. So, I looked and looked for another line of work within the system until I found the new technology job I currently have. (Jones, 2005, p. 64)

The National Broad adventure armed Laura with new knowledge and confidence. During the process Laura forged a new identity as an educator. Laura started to reflect on teaching and provided advice for schools and teachers. She began to support teachers, in particular new teachers', growth by recognizing them as learners. Laura told:

> At this time, teaching is viewed correctly by many young people as a no-growth profession. But that perception could easily change if school systems saw their teachers and other personnel as people who have multiple skills and talents to offer. (Jones, 2005, p. 64)

Getting National Board Certification shaped and reinforced Laura's identity as an educator. This experience informs her teaching in GEMS that integrates technology into GEMS activities. Through GEMS efforts, she continues to inspire teachers to develop professional knowledge and encouraging girls and minorities in STEM fields.

Lili: Study Abroad

As self-awareness grew during teaching, I began to think about students as subjects of learning. The school where I taught was an urban middle school. Many students were *floating children* (i.e., children from rural or suburban contexts brought to the city by their migrant parents who sought job opportunities). They often had interrupted educational experiences, moving with their parents from here to there, and facing challenges fitting into urban schools (Mu & Jia, 2016). Without legal residence identity, the floating children could not register in the school system as local students. They were allowed only to attend compulsory education (i.e., Grades 1- 9) and had to find high schools in other places.

I learned the meaning of teaching from the work with my floating students. Teaching is not teaching a subject but teaching whole people who come from the past and move forward to the future. Along with being a mathematics teacher, I served as a Grade Department Chair who worked closely with students, other subject teachers, and parents. I had recognized that without a positive identity in learning, students would be less motivated, thus wasting the time of both students and teachers. I noticed that being concerned about academic performance was not enough, in particular, for many floating children who could not see hope from education and were not interested in academic performance. Working with floating students and their parents, I learned that, besides academic performance, we also need to develop students' positive dispositions and identities in school. However, examination-oriented education focuses primarily on students' performance and grades, which are even associated with teachers' teaching performance. Thus, the floating children were often underserved, resulting in less confidence in school.

One summer, my students were taking high school entrance examinations. As the Department Chair, I hosted lunch with the students. During lunch, I talked with a group of students about their plans after they graduated from middle school. All the students except for a quiet girl had targeted specific high schools. The girl drew my attention. She told me that she did not have a resident identity in the city, so she was not allowed to attend high school in the city. Her parents left their hometown 20 years ago and did not have any connections with their hometown; it was also not easy for them to find a school in their hometown. From the conversation, I learned that the hometown of the girls' parents was close to mine, where I have many connections with teachers and administrators. I told her I would like to find a high school for her. After the examinations, her parents reached out to me and asked for help. I helped her find a school where the girl completed her high school education and went on to enroll in a reputable university. She often sends greetings to me on Teachers' Day, a national holiday in China which celebrates teachers. Currently, she works for a company in Beijing and is saving money for graduate school.

In recent years, Beijing has implemented control of rural migrant population policies that limits people from other places to immigrate into the city, which have significantly restricted floating children's access to public schools (Zhang et al., 2017). According to the National Bureau of Statistics of China, in 2016, the population of floating children in China who graduated from middle school decreased from 2,490,760 in 2015 to 918,719 in 2016, about a 63% reduction (China Statistical Yearbook, 2016). Since 2016, every year the number of floating children graduating from middle school has been less than 1,000,000.

In my research, I did not find much information about the floating children from literature. From my anecdotal experiences, not many of my students who were floating children had the opportunity to attend college. Migrant families often send their children back to their hometown, far away from their parents, staying with their extended families or attending boarding schools. Many of them do not have a strong connection with their hometown and struggle to adjust to school there. Many talented students leave school after a short time and return to the city. They either attend training courses or directly find jobs and do work similar to their parents. Thus, many underprivileged children who do not have access to higher education continue to be marginalized just like their parents. I became frustrated and felt discouraged about the impact on these students.

For students who go to college as first-generation college students, their experiences are often like mine. They do not know what resources are available to them and how to seek out opportunities. Some previous students have reached out to me to ask how to apply to graduate school and change their major. I am always happy to provide this information. For me, teaching involves building lifelong relationships between students and teachers, supporting students' growth, and promoting their self-development. I discovered that the meaning of teaching motivated me to learn more to support my students.

As a teacher, I have firsthand evidence of the struggles that migrant families and children face. The phenomenon motivated me to rethink my career and either continue being a politically neutral teacher or expanding my knowledge to influence underrepresented students. I decided to pursue a doctoral degree. I first looked at universities in city in which I was living. However, I found out that I could not apply for a Ph.D. program because most programs required that you were under 35 years old. I was angry; why could a woman in her mid 30s not have access or the opportunity to change her life? People told me that there is no age requirement in the United States. I spent two years preparing for an unknown journey to change my role from a classroom teacher to a teacher educator. The training I have received in my doctoral program prepares me to listen to the voices of marginalized people, speak for them, and make a substantial impact in teaching.

Chapter Summary

In this chapter, I explored Laura's family stories and personal narratives and shared my own family histories and personal narratives of being and becoming a woman, a teacher, and an educator. By telling and retelling these educative experiences, I expressed them through stories as a mode of knowing that creates our personal practical knowledge. I developed personal practical knowledge that illuminates our understanding of teaching, learning, and justice in education. I also explored our personal and professional identities across contexts. The narratives of our experiences illuminate how multiple identities have evolved as a source of tension, resilience, and transformation that continue to inform evolving personal practice knowledge. In the next chapter, I explore GEMS as an informal learning space empowering girls in STEM. Through exploring stories in the setting of GEMS, I continue the inquiry of our personal and professional identity as female educators.

CHAPTER 5. IN THE MIDST OF GEMS

GEMS clubs vary depending on how leaders structure their own GEMS clubs and what curricula they provide. For example, I recently observed Laura's GEMS club in which girls were working on an engineering project in a science lab. The setting was similar to that of a classroom, but the atmosphere was informal. Girls could chat, make jokes, and move around. First, Laura gave girls instructions and then girls worked on their projects. The girls asked leaders any questions they had and discussed their projects with peers. Laura and other leaders walked around the lab and provided feedback on the girls' work and asked questions. When Laura noticed something related to project and needed to discuss it with the girls, she brought them together and gave them instructions. Then, the girls would continue their work.

In the summer of 2021, I led a GEMS club in my neighborhood. Instead of a school setting, my GEMS club took place in an outdoor setting in the community. It was summer and the weather was nice. Children ran around in the neighborhood, and I saw they were bored, which made me think of gathering them to do some activities. From my viewpoint, I wanted them to learn something from playing. I recalled my own experiences when I was a child and which games and activities were interesting to me. I asked my friends in Physics and Science about interesting activities for younger students in their fields. I modified 10 collected activities I felt would be the most interesting to the girls. I met the girls at an outdoor area and used picnic tables as our workplace.

Two days before the day of my first GEMS club, with my own daughter's assistance, I informed girls in the neighborhood that we would meet at 10 am outside. When the day came, I organized materials and supplies. Through the window, I saw girls were looking in my direction and waiting for me. They were excited to see me come out and bring the materials. They helped me set up groups and distributed materials. They touched the supplies with curiosity, "Ms. Lili, what is this? Is this sugar? Why did you bring eggs? What are we gonna do?" Sometimes, I provided an overall description of the activity first, then they started to explore. Sometimes, I gave instructions during the exploration. The girls did not see me as a teacher but rather as a parent and a neighbor. We talked about math as if it was everyday conversation.

GEMS provides a context in which Laura's and my collaboration is located. Along with this exploration, Laura and I work together to move the GEMS program forward to address gender injustice in STEM as well as in society. In this chapter, I first tell stories of GEMS as a learning space in a historical, social, and political context. Then, I explore the leaders' stories in GEMS. I also reflect on my own experiences in leading learning in informal environments. By telling and retelling these experiences, we are developing a clear and deeper understanding of ourselves as informal educators. Finally, I describe GEMS as a space that empowers girls and women in STEM, and I explain why we need learning environments like GEMS to inspire women. Working in and for GEMS allows both Laura and me to look to the past, current, and future to unpack our stories and the contexts of those stories, which have shaped our identity as informal educators. We also have an opportunity to look inward to question our inner voices and to look outward to challenge the patriarchy in mathematics/STEM fields.

Knowing GEMS as an Informal Learning Space

Laura describes GEMS as an informal STEM program that provides an alternative to classroom learning for girls. GEMS has been providing a space for Laura's daughter and other girls—as well as for Laura herself—to become the persons they want to be. Laura emphasizes that GEMS is different from school; it focuses on learning through hands-on activities. Teaching in GEMS is self-regulated, as opposed to following a specific curriculum and requiring standardized tests. Both Laura and I conceptualize GEMS as an informal learning environment, providing STEM educational opportunities for girls. In the first subsection, I tell the history of GEMS, why Laura started GEMS, and the nature of the first GEMS club. Then, I explain how GEMS as a learning environment is different from school settings. Last, I describe how learning occurs in GEMS. I intentionally use Laura's quotes and her own narratives from journals, interviews, and GEMS Handbook to present her voice. Through telling and retelling, I intend to reveal her beliefs and identity as an educator.

Starting GEMS for Girls

People are curious about what motivated Laura to start the GEMS club and how she operated the program for 28 years. Laura wrote:

> In 1995, at the open house for the new science/technology magnet school where my ten-year-old daughter had been invited to attend, we joined other parents and students for an exciting two-hour session observing and participating in computer

> simulations, science demonstrations, and language arts projects. Afterward, my excitement about the coming school year was dampened by the realization that every time [Janet] went to try one of the math/science/technology activities, she had been pushed out of the way by the boys or had not been called on by the group leader. She had not been willing to try the new activities, and she, too, was aware of her reluctance. Back in the car, she said, "Math is hard, Mom." I felt like I was talking to the recalled Math Class Barbie doll. (Jones, 2002b, p. 1)

Laura recalled her reaction when she heard Janet's complaint that "math is hard", "What's going on in your head that makes you think math is hard? Because you're not—you're good at math, and that was really frustrating. And then—so then it became a project" (Laura, Interview, October 2019). For Laura, mathematics was essential for Janet to become successful in the future. She could not accept that Janet gave up on mathematics. At that time, she said to herself, "I wasted my opportunity. I've got to raise my kids, so they don't turn down the opportunities." This idea drove Laura to start the first GEMS club in Janet's school, which provided a way for her to be involved in her daughter's education.

I first heard this story when I met Laura at a meeting with mathematics education faculty and some graduate students who were curious about how she started the GEMS club. Later I interviewed Laura in a hotel meeting room where she retold the story in detail. I could feel her disappointment in Janet even though the conversation had happened many years ago. Janet did not end up attending the magnet school, which motivated Laura to discover what had happened and determine what she could do to help.

It seems like just a random opportunity that caused Laura to begin the GEMS program. In one interview, Laura told me that the GEMS program was important to her because she felt as if she was not born at the right time, which prevented her from becoming the person she could have been. She said:

> This happened in April. They opened up the magnet school. I said, "Janet, you should go to this. This is so cool." This was a partnership between Fairfax schools and the Smithsonian [Museum], if you can imagine. The Smithsonian came into this school to work with these kids, and she's like, "No, I'm not going to go." So that was like—we've got to figure out something to make an opportunity that she has to do. "I started a club at your school, and you have to go." . . .. Then it became my project, to be really honest. I love this [GEMS]. It added the whole thing about girls' importance, gender equity, I could have been an engineer, all into one little pile related to education. You see, it was like the perfect thing for somebody who felt like she was born too late—or too early. I was born too early to become what I could have been, and so, by God, I'm not going to let that happen to other girls. (Laura, Interview, October 2019)

During that summer, Laura read educational research reports and learned many girls had a negative attitude toward mathematics and science; they did not want to take high-level mathematics classes and often had less confidence in mathematics and science. The research findings resonated with Laura as a parent and she realized in addition to her daughter, many girls struggled with mathematics and science. Having received a university education, Laura clearly knew that if students did not complete enough mathematics and science courses, their future career choices would be limited. They would have great difficulty entering many scientific fields leading to innovative career path and higher social status. Laura wrote:

> I knew that I had to take action. I did extensive research on girls' attitudes towards math and science. I brought the Legos out of the basement, and I stopped saying I was bad in math. I also started an afterschool club for fifth- and sixth-grade girls at my daughter's old elementary school. (Jones, 2002a, p. 3)

Laura realized that she had not provided a good education and an enriching home environment for her daughter, and that the school was not doing enough to support girls in developing confidence and interest in mathematics and science. She decided to provide opportunities to help girls like her daughter see themselves as being successful in mathematics and science at an early age. With this motivation, she talked with Janet's teacher, Ms. Cooper, about the idea of starting an afterschool club. Ms. Cooper, who had more than 20 years of classroom teaching experience, had witnessed girls' low participation in mathematics and science. She enthusiastically responded, "Absolutely!" Together they convinced the principal to approve the GEMS club. Laura said:

> I admired her [the principal] for that because that was risky, and she knew that she might have parents calling her or they would call the school board. . .. The school placed us under the umbrella of the PTA-sponsored afterschool program that provided transportation. (Laura, Interview, October 2019)

Laura described the rules and activities of the first GEMS club:

> My co-leader and I had a few unwritten rules: no men/boys involved; food was always present; and the activities would be fun, not standards-driven. . .. The girls who enrolled were always mixed in age. Because we felt strongly that this should not be remedial or a gifted program, we had a wide ability range. (Jones, 2002b, p. 3)

As I heard the story Laura told, I was curious about how GEMS works in school but not like school and how Laura as a parent contributes to education in such an influential way. From my experiences as a student, a teacher, and a parent, school dominates students' educational lives.

Parents can offer additional support to supplement school education, while continuing to promote the school's mission as the primary educational experiences for students. More than anything else, curiosity motivated me to learn more about Laura's experiences with GEMS and her contexts.

GEMS Is Not School

Twenty years ago, Laura wrote how she conceptualized GEMS:

> Over the years, my goals crystallized and I was better able to articulate them. I wanted the girls to try new, hard things and to try them in a place where they would not experience failure in the typical school sense but would know that it was part of growing to take risks and make mistakes. (Jones, 2002b, p. 3)

Recently, in describing GEMS, Laura emphasized, "The bottom line for GEMS is that it is not like school!" She further added, "This doesn't mean I am against school; I am not." She explained that GEMS is different from school in many aspects. For instance, in science classes, materials and supplies often are used only in demonstrations; in GEMS, enough supplies are provided for every girl. Classroom learning is often solo and focuses on individual development; in GEMS, the learning environment is more collaborative and less competitive, so girls can feel free to take risks. More than a learning environment, GEMS also provides social space for girls to invite their friends to join in and make new friends in which.

The most important characteristic of GEMS is that, in contrast to following a set school-approved curriculum, GEMS does not adhere to a specific curriculum. Instead, leaders choose their own curriculum. Laura does not want the leaders to feel obligated to use a specific set of activities. "I don't ever want to say to people [leaders], 'You have to do these activities,' because that takes the joy out of it."

In a journal, Laura described activities from the first GEMS club and explained how she and her co-leader selected the activities:

> We chose activities we thought would be fun and interesting and used whatever resources we could find. My favorites came from *Scientific American* and from Mensa publications. My co-leader drew from her many years of experience and her attendance at the Virginia Science Museum weekend. We explored geometry with hedrons and topology with mobius strips. We explored our own mazes and codes. We participated in the internet M&M survey and in the Estimation Olympics. We learned several number games and strategy games and we explored optical illusions. . .. The second year included activities in chromatography, culminating in making GEMS shirts with chemical reactions. We had a guest speaker—a female

cartographer—and a map scavenger hunt. We did an estimation activity with drops of water on pennies and experimented with movement and flight with rocket pinwheels and balloon trolleys. (Jones, 2002b, p. 2)

Laura always emphasizes that the primary goal for learning in GEMS is enjoyment, for both leaders and girls. In addition, leaders must come to GEMS with ideas that they are excited to try. If they do, girls will also be excited. Because there is no predetermined curriculum available for GEMS, the leaders have to search for resources and explore learning opportunities from the activities. In this regard, compared with classroom teaching, leading GEMS is more challenging for leaders, who must design activities that will engage girls. On the other hand, leaders have more flexibility than classroom teachers, in that they can construct their own curriculum. Laura described her satisfaction as a GEMS leader:

> When it [the GEMS session] ended, a feeling of satisfaction that the girls were able to enjoy hated subjects through creative activities came to me. For the rest of the year in GEMS, I received that feeling every time it ended. (Jones, 2002b, p. 2)

Laura encourages leaders to choose activities and tasks that are ahead of the girls' current grade level — "something that girls will do two years later." Laura stated two reasons that drive this decision: "Girls will think it's cool two years later when they know how to do it, but it also presents a challenge." Girls feel excited by learning advanced knowledge, which, in turn, helps build their confidence. From Laura's perspective, because the GEMS format is informal, it is unnecessary to follow a particular curriculum or sequence of activities. Thus, leaders have the autonomy to select and implement their own curriculum, which makes GEMS unlike school.

Many times, some Purdue GEMS team members have raised the issue of the lack of a set curriculum, which has created some tension among its members. The curriculum is a critical component of learning and a primary tool for teaching. As a space where learning happens, it is a part of curriculum; in turn, the curriculum should reflect the mission of GEMS. Though many activities and lesson plans are provided on the GEMS website, leaders have the freedom not to choose these activities. However, without a particular curriculum, some teacher educators may find it difficult to describe GEMS as a learning environment.

During the collaboration with the Purdue GEMS team, Laura started to think more about curriculum. Recently, Laura expressed her concern:

> I'm hoping we can do some research on integrated STEM. And maybe we can figure out how to reframe all the activities and theories and thought processes. We

> have to maybe do more of a curriculum for certain age levels, rather than by subject. I mean, we're gonna have to think about it. (Laura, Interview, June 2021)

In particular, Laura integrated curriculum, rather than teaching individual STEM disciplines. Building on her GEMS experiences, Laura provided suggestions for classroom teaching: "In elementary classrooms, making classroom computers into centers with assigned times and activities gives girls and English-language learners equal footing with boys to experiment and explore, building confidence." (Jones, 2008, p. 10)

School learning has clear and specific goals and learning outcomes to be measured, whereas GEMS' primary goal for learning is to engage students and develop their interests and identities. Laura uses reflection cards to provide girls with opportunities to think about their participation and learning. One set of reflection questions focus on the girls' collaboration in group work, such as, "How did you contribute to today's activity?" "How did your group work together?" "How could your group be more effective?" "What problem did your group solve today?" "How did another person in GEMS show collaboration today?" (GEMS Handbook, 2019, p. 62). Another set of reflection questions aims to develop the girls' disposition, such as "Why did you succeed today?" "What risk did you take today?" "What was the most challenging thing you or your group did today?" "Did your group become frustrated?" "What did you learn about yourself and your friends?" (GEMS Handbook, 2019, p. 64).

Laura justified GEMS having a learning environment different from school by saying:

> If somebody said there was no accountability, I would not agree. There is accountability in the girl. Did she leave GEMS learning something? And there was accountability, of course, in the leaders. Did I get across? Did I give them the kind of experience I want? So there is accountability. I think it's something really different because there's so little opportunity in schools to do anything that is not encapsulated in some standards. That's why I want a different [from school]. (Laura, Interview, October 2019)

Learning and teaching are redefined in GEMS. In addition to focusing on the development of content knowledge, GEMS aims to promote girls' engagement in learning and leaders' autonomy in teaching.

GEMS as a Learning Environment

Laura and I both participate in the Purdue GEMS team. As a core member, Laura brings her insider view of GEMS, resources, and network to the team, which helps the team to better understand GEMS. I, as a novice researcher, participate in GEMS practical work, such as meeting current or potential GEMS leaders, organizing GEMS leader workshops, and seeking new resources. To better understand GEMS and Laura's experiences, I explored the informal learning literature; that is, what is informal learning, including how has it been conceptualize and studied. I have conceptualized GEMS as an informal learning environment where learning and teaching have been redefined without being bound to a curriculum. Though GEMS often takes place in a school setting (i.e., classroom or lab) and many leaders are also K-12 classroom teachers, the program does not necessarily address a particular subject or specific content knowledge. Laura sees GEMS as an informal learning space that is alternative to school learning.

Informal learning does not have time and schedule constraints. This flexibility makes GEMS a designed informal learning space wherein learners are approached as whole people. The learners participate in the GEMS activities, interact with each other, and construct knowledge in a casual mode. The leader adopts a loosely planned learning goal, embraces unexpected ideas, and allows such unexpected ideas to lead both learners and the leader to an unknown situation. Laura refers to the unexpected moments as opportunities that potentially develop students' dispositions, which she sees as a source of motivation. She expresses that the content knowledge can always be taught but critical moments that stimulate wonder and curiosity are allowed to emerge, even sometimes these are seen as irrelevant to the topic.

According to Laura's (2002a) unpublished manuscript, in 1997, the third year of the first GEMS club, Laura conducted action research to investigate the impact of GEMS on the OGGs' attitudes toward mathematics and science. She collected data through surveys; interviews with the girls, their classroom teachers, and the parent-child team; and a written log of her own reflections. Laura received feedback from classroom teachers and parents that indicated positive changes in the GEMS girls. Eliza, an ESL student, rarely spoke in the classroom or GEMS, but she demonstrated her ability to perform well in the GEMS activities. Eliza won a strategy game in GEMS, which was reported in the school newsletter. Soon she began to exhibit more confidence and to excel in her ESL class. In another example, a mother shared that her daughter had shown dissatisfaction in her ability to solve problems, but after participating in GEMS, her daughter's confidence had increased, and she showed less frustration. Other parents also provided positive feedback about their daughters' participation in GEMS. Overall, the girls reported that they enjoyed the GEMS activities and attending the group had positively influenced their learning of mathematics and science in the classroom (Jones, 2002b).

Five years later, in 2002, Laura once again conducted research with the same group of parents and the OGGs to investigate their course taking and career interests. Even though the OGGs had not yet determined their career directions, they still saw the value and impact of GEMS in mathematics and science and other courses like technology. OGGs' enrollment in advanced mathematics and science courses was significantly higher than the average enrollment in the county. For instance, the enrollment of girls in the county in Pre-Calculus was 3.1% which the enrollment of OGGs was 12.2% (Jones, 2002a, 2002b). Laura wrote:

> I think that this club and the attitudes we instilled in these girls must have made a difference. There is no real statistical data here—the samples are too small. But clearly, a higher percentage of the GEMS girls chose to enroll in more advanced classes than the average girls in the county. And they did not do this with particular careers in mind but seemed to see the inherent value of the challenging courses with no immediate gratification. My goal was to see that girls did not place limits on themselves. I think that it was achieved. (Jones, 2002b, p. 19)

In 2020, with Laura's assistance, the Purdue GEMS research team contacted 14 of the OGGs and conducted interviews with nine of them to investigate the long-term impact of GEMS on their attitudes about mathematics and their career choices. Many of the OGGs have long-lasting memories of their experiences in GEMS. Some were able to name GEMS activities and share their memorable learning experiences. For instance, Anna, one of OGGs responded to our survey: "I remember very specifically making the bridges out of toothpicks and gumdrops and the rockets out of Alka-Seltzer and vinegar." Even those that did not remember specific activities expressed strong, positive memories of enjoying the activities and being excited about learning because the activities made it fun. In the survey, another OGG, Courtney shared: "I can't remember any [activity]. I remember the feeling of being excited about the projects." OGGs also appreciated the group work and collaboration opportunities in GEMS.

The OGGs described how GEMS influenced their understanding of mathematics and attitudes toward learning, parenting, and career directions. Stella, one of OGGs shared:

> I don't remember a ton of specific activities, but I do remember that it [GEMS] made me less afraid of math. Coming out of third grade where I always failed my times table quizzes and tests because I just didn't understand the concepts, GEMS made me think that math wasn't as scary. I learned that I understood applied math in a way that I didn't get from memorizing times tables. (Stella, Original GEMS Girls Survey, 2020)

The longitudinal studies (i.e., Jones, 2002a, Zhou et al., 2021) indicate that GEMS offers a learning environment that is an alternative to the school curriculum. Learning is redefined in GEMS. Laura shared the secret of GEMS: "In GEMS, they're going to learn, but we're not going to let them know they're learning." Laura further explained, "What I want from them is excitement when they leave." The role of teachers in GEMS is also redefined. GEMS provides a space for leaders as curriculum developers, to make learning interesting and fun as their primary goal.

GEMS Leaders as Informal Educators

In this section, I first share stories of two GEMS leaders who have led or currently lead GEMS clubs, drawing on meeting notes and my own reflections. Although this study does not focus on GEMS leaders, their stories shed light on who GEMS leaders are and what they do in their clubs. Then, I use my own cross classroom and informal setting teaching experiences to illustrate the opportunities and challenges that exist for a classroom teacher who wishes to become an informal educator. Finally, I further explore Laura's expectations for GEMS leaders.

Stories of GEMS Leaders

Laura regrets that she did not maintain a record of GEMS leaders. The GEMS website is published as a virtual library free to the public, which makes tracking leaders more difficult. Many clubs adopt the name and mission of GEMS, doing STEM activities with girls, but they do not necessarily use the website resources. From the network, I have learned that GEMS leaders consist of K-12 classroom teachers, parents, high school girls, and female undergraduate STEM majors. The GEMS website displays a registration page and encourages people to register on the site. Through the registration form, we have begun to build connections with GEMS leaders. We attempt to contact those who need support or have questions. For instance, we have received an email from a schoolteacher, Hanan (a pseudonym) who wrote:

> I am thinking of starting a club next school year. It depends if I am placed up at the high school or not. Currently, our elementary school does Girls Who Code, but there are no STEM options available at the high school level. What information can you provide me? Are there any grants that are specific to GEMS that would give funding to do projects within the club? (Hanan, Email, March 2022)

Based on Hanan's questions and concerns, Laura responded her:

> That's great! I would recommend that you start by downloading the Handbook from the web site. As far as grants, start with your local community foundation. https://www.facebook.com/belprefoundation/ Their grants are open right now! And ask your principal/superintendent if the schools have community partners. Are there any big industries nearby? Any big companies? I see that you have manufacturing companies. They are always looking for STEM people. Also, the two colleges Marietta and WVU might be willing to sponsor something. It takes time. Your foundation is the best bet right now. Let us know. We can help you solve problems. (Laura, Email, March 2022)

Through building connections with GEMS leaders, I have opportunity to listen to their stories and experiences with a passion for understanding of GEMS. Here, I will share two leaders' stories: Ms. Cooper, a classroom teacher who was Laura's co-leader of the first GEMS club, and Danae, a mother and mathematics lecturer at a post-secondary educational institution.

Ms. Cooper

Janet's teacher, Ms. Cooper, and Laura worked together to develop the first GEMS club. Laura wrote:

> After my August request to the school to start and develop the club, a teacher immediately volunteered to help: my daughter's energetic, experienced, fifth-grade teacher. Apparently, she had been waiting for the opportunity to do something like this. Four years later, she said, "I remember quite well when you brought up the idea and I loved it. I was very busy with lots of school stuff, but I felt like I just had to find time to do GEMS because it would be one way to help girls with a little extra boost that I have known forever that they needed." (Jones, 2002b, p. 1)

At the time of the first GEMS club, Ms. Cooper was an experienced teacher who had taught in the classroom for about 30 years. In the spring of 2020, with Laura's assistance, I was able to interview her. Retired for 20 years, Ms. Cooper's memories of GEMS were somewhat vague and overlapped with her classroom teaching. She told me her school supported afterschool programs at that time; many of her colleagues were offering afterschool programs—some in their classrooms and others in various school areas. She recalled her memories of how she came to be involved with the GEMS club:

> When I was young, I envisioned myself teaching English and history and that sort of thing, but as I got into teaching, I found out that most of us were women, and most of us liked to teach English and history. So, I was being the kind of low man on the totem pole, being young, I started doing math and science and found out I enjoyed math and science quite a lot, so I just continued in that vein. But I also noticed that girls weren't really wanting to be that involved in math and science.

> And so this was a way for me to introduce some fun things that they might not get in a normal classroom setting that we could do together in afterschool, and it would be just for girls. . .. Those girls were in the 10-, 11-, and 12-year-old range. So that's how it got started, and these girls were gems, bright, shiny little stones, but possibly they would see another side of math and science that they hadn't seen before. And they liked me, so I always had a full group signing up for this afterschool special. I did do more than just math and science. I would try to have a female who was out there in the working world doing something come in and talk to the girls about their careers, too. So that was just kind of another facet of the time that we'd have together. (Ms. Cooper, Interview, February 2020)

Ms. Cooper said proudly, "It [GEMS] was before anyone came up with the term of 'STEM.' It just turned out to be a little bit ahead of its time." She was excited to know that GEMS continued after she retired.

In our recent study with OGGs, several OGGs mentioned Ms. Cooper about how she impacted their learning in mathematics and science. Kim shared:

> I don't really have any particular memories of the GEMS club, but I do remember how much I loved Ms. Cooper. She was definitely my favorite teacher and one of the few I can actually remember today. I remember how supportive and encouraging she was. (Kim, Original GEMS Girls Survey, 2020)

I shared the OGGs' memories of Ms. Cooper through emails, she was satisfied with the comments from the girls after 26 years.

Danae

When we learned that Danae had led a GEMS club at some point, the Purdue GEMS team contacted her and scheduled a meeting in which Danae shared her experiences with us. Ever since she was a high school and college student, she had been interested in increasing the number of women in mathematics and computer science. Her undergraduate degree is in science, with a minor in mathematics and women's studies, and she conducted research with women in computer science. She obtained her teaching certification and taught high school mathematics and AP computer science, after which she took a break from her career to raise children. I first met her in 2018. At that time, she had four boys and a three-year-old girl (the youngest); she also had a part-time job teaching entry-level mathematics courses at a post-secondary educational institution.

Danae's husband is a computer science professor; he also coaches the National Science Bowl at the local middle school. Danae's husband had explained that hardly any girls participate in Science Bowl, which inspired her to reach out to girls before middle school to get them involved in science and mathematics. Danae assumed that if middle school girls were involved in science or mathematics earlier, they would continue to take more STEM courses in high school and even post-high school.

When Danae researched the topic of girls in science, she located the GEMS website and loved the club's mission and name. She proposed starting a GEMS club for third through fifth graders with the goal of getting them to continue in science courses. She called her girls "Rubies." The girls enjoyed science and doing experiments, and Danae received positive feedback from the girls, their parents, and the teachers. She found that her GEMS girls did not like mathematics, nor did they have confidence in mathematics. Thus, she planned to focus on just the math component in the spring semester with girls who were interested.

Surprisingly, Danae started up the GEMS club as a parent at her sons' school, but she did not involve her sons in GEMS. She told us:

> I have all boys at that school and my boys wanted to do it, but I said no, absolutely not. Because girls like being in an environment where they don't have to feel like the boys are going to talk over them, that they don't have to feel concerned about their idea not being [heard]. (GEMS Meeting Notes, 12/6/2018)

Danae's identity as an informal educator in GEMS does not align with her identity as a mother of boys. She does not struggle with that tension; rather, she values a separate space for girls and wants to continue supporting girls in mathematics and science.

Leading Informal Learning Is Challenging for a Classroom Teacher

As I engaged in GEMS work, I recalled my own experiences in leading informal learning when I was a mathematics teacher in China. For an entire year, I led informal learning—one semester for a Paper Airplane Club, and one semester for a Modeling Club. However, I have always introduced myself as a former classroom teacher and have never thought to mention my experience in leading these clubs. Though the two clubs were in a school setting, the content taught was not like school subjects; there were no quizzes or tests, but final products were expected.

I chose to create a Paper Airplane Club because as a mathematics teacher, in my geometry teaching, I often used paper folding to illustrate the geometric relationships among points, lines, and angles. I expected students to develop geometric intuition through manipulations. In addition, I tended to provide a space for students to have experiences that were both fun and stress-free after a whole day of intensive classroom learning. I introduced basic origami geometry to the club and provided materials and some models of paper airplanes. Students were allowed to design their own models. I opened the club to all students but, was surprised when only boys joined the club. Some of them excelled in academics, whereas others were not so successful. Nevertheless, they were all welcomed in the club and actively participated in the design and testing processes.

Unlike the Paper Airplane Club comprised of only boys, half the students in the Modeling Club were girls. I introduced the attributes of different geometric objects and provided a set of K'Nex (i.e., a rod and connector building system) with a variety of instructions for suggested models. When the club began, I suggested students choose some of those models they would like to build. After they became familiar with the tools, they were allowed to design their own products.

For both clubs, each lesson presented during the semester was carefully planned and structured. Unlike the classroom, the atmosphere was relaxed, and students were not ranked according to their performance. I did not position myself as a traditional role of teacher, because, when designing models, I did not necessarily know more than the students. Most of the time, I walked around the room, joined their groups, and asked or answered questions. The students also saw me as a supporter rather than a mathematics teacher in the club. They did not ask me how to solve a problem that I had assigned. Rather, they had clear goals and asked me for help to achieve their goals.

Because the nature of the club was informal, the emergent questions were not limited to mathematics. I felt limited by my specialized knowledge and skills in mathematics. Students were excited to explore new things at the beginning of the club, but as they became familiar with the process, they could easily become bored and lose interest. I needed to present innovative tasks in order to inspire students to continue to be passionate about learning. I found it very challenging because there was no existing curriculum to use, and I also was not experienced in developing non-mathematics curricula.

I faced other challenges as well. First, I did not develop assessments for the clubs, thus, I consistently questioned whether students were indeed learning during our meetings and whether the club's structure was effective in inspiring students to learn subject matter in their classrooms. I also struggled with my role as a leader, since I realized my knowledge in the clubs was limited. Without more knowledge, I felt I was not an authority among the students, and I lacked confidence in front of them. Thus, my self-efficacy was affected, and I questioned my qualifications in that space. Additionally, because the school and teachers focused more on improving students' content knowledge to help them succeed in examinations, my colleagues viewed a club leader, like me, as someone who "plays" with students rather than guides learning.

In the first couple years of leading the GEMS club, Laura also encountered many challenges which she called "frustrations." (Jones, 2002b, p. 3). First, she felt that girls working on the activities took all the time and did not leave time to discuss and reflect on learning. Second, in GEMS, Laura was Janet's mom rather than a leader or a teacher, Laura felt girls did not listen to her and follow her. She said:

> I was nervous and the girls knew it. I had no real identity during the first year other than "Janet's mom." I always felt that the girls didn't really listen to me. But that improved over time as I chose better activities and as the girls saw me every week. I gave them credit for their patience. And I'm glad they stuck with me. One moment I made in my log: "am I just a mother and therefore suspect?" (Jones, 2002b, p. 3)

At first, Laura was not confident about her classroom management skills, but later she realized that through choosing appropriate activities, she could manage learning in GEMS very well. Over the time, she did not question her qualification of being a leader in GEMS and got more and more confidence in practice.

A Good Teacher May Not Be a Good GEMS Leader

In the GEMS Handbook, Laura described and advised GEMS leaders:

> That would be you. You are the leader, the person or persons who are responsible for organizing and planning all of the activities, the person who spends nights trolling the Internet for fun science and engineering activities, and weekends begging for supplies and donations.
>
> The most important thing you can do is be sure to have fun. That is what the girls will remember. Even if the experiment doesn't work, or the towers they build fall off the desks, your attitude toward the activity is what they will remember.
>
> You don't have to know everything. Keep a running list of questions on the board or a large piece of paper and commit to finding the answer to them.
>
> You are the planner. You have to be the one who keeps things flowing. Deadtime is not good with children; you can lose their interest very easily. So, plan and make a list of things to do in order. (GEMS Handbook, 2019 p. 33)

Based on Laura's descriptions, I understood a GEMS leader to be a person who is passionate about seeking new resources, trying new things, and exhibiting bravery when faced with failure in front of the girls. In an interview, I asked Laura, "In your experience with GEMS, what kind of knowledge, skills, beliefs, or attitude do you think a GEMS leader should have?" "Oh, boy, I could go on and on about this," she laughed. Then she said:

> They need to be willing to experiment and try new things. And they need a sense of wonder. Most teachers still have that. Look, at my age, I feel like I still have that [wonder]. A really good teacher never loses the idea of "that is so cool!" (Laura, Interview, June 2021)

She added, "And they have to be able to look at themselves and say, 'I could do better.' It's what every good teacher should be."

Laura further pointed out, "While a good GEMS leader is probably also a good teacher, a good teacher may not be a good GEMS leader, because she may not be willing to let things fail." Classroom teachers are prepared for teaching subject matter, and they are expected to be experts in the content they are teaching. However, time limitations and a focus on content and subject knowledge may prevent classroom teachers from being able to capture evolving learning opportunities. Laura stated, "It's not like informal learning has to be always structured; it's the 'never miss an opportunity.'" To illustrate the meaning of "opportunity," she described a scenario when she visited a GEMS garden club:

> Somebody was saying, "Look what happened. We turned over last year's mulch, and there were milkweed plants. It was so cool! So, we dug out all the mulch. Now look, in two days, they're starting to grow again." Then one other kid said, "Yeah, I looked online. What butter . . . who knows? Not butterflies, like milkweed. One of the insects likes milkweed. That's what they like. So, we're going to start watching to see if they start to come." And then somebody said, "I wonder if we could set up a webcam out here." Instead of saying "No, that's too hard; it'll rain," the leader said, "Hmm, I don't know; I'll talk to the tech guy. Let's think about where it should be." You see what I mean? It's the difference in immediately saying no to the opportunity. "I don't think we can do it. But we'll see if we can." It's that opportunity. That's kind of a long, messy story. But that's what, to me, informal learning is about. Maybe they'll have a webcam out there at some point. And that's because some girl suggested it. That it's like taking that opportunity and running with it. And informal learning can do that more easily than in a classroom because you do have a curriculum you have to get through. (Laura, Interview, June 2021)

Unlike classroom teachers, informal leaders have more freedom and time to explore with their students; rather, a willingness to learn is the most important characteristic of informal educators.

Some GEMS leaders are not confident in mathematics, but leading GEMS is a way for them to learn together with the girls.

In the fall 2021, Laura applied for a grant to support her purchase of an engineering-based STEM curriculum. She implemented the curriculum in a GEMS club with two groups of seventh grade girls. Through an online meeting platform, I have observed Laura's teaching in her GEMS club. On one occasion, she introduced the concept of scale in construction to the girls as they began a carpentry project. She was honest that she was not familiar with the problem but wanted to figure it out with the girls. Later, together they celebrated their successes and reflected on their failures. The girls were very excited to see that Laura, as a leader, took part in the learning process.

Laura shares concerns that too much emphasis on content knowledge might make learning less engaging. She said, "They [educators] can know everything in the world, and the kids will hate it. And by hating what they do, they're going to hate science, or something [math]." Leaders need to get out of their comfort zones (i.e., specialized areas) and reach across discipline boundaries to engage knowledge in which they are not confident.

Knowing GEMS as a Space to Empower Girls

In addition to the informal learning feature, GEMS also is a girls-only space. Laura started GEMS for girls, or learners who self-identified as girls. Since the first club, people have questioned the rule of excluding boys and male leaders, which may be another form of injustice. In an interview, Laura said:

> For 26 years, I have been justifying this. It isn't me. It's the idea; it was the idea I think the core value is that girls have value. The core value that girls can learn. Girls are just as important as boys. It's not boy bashing but let's get real. Boys start up here [she gestures up high] and girls start here [indicating a lower point]. I wish GEMS could make that a little equal. (Laura, Interview, August 2019)

Throughout this narrative inquiry I have consistently reflected on the reason we need this space for girls and whether this space can make a difference. In my dissertation proposal defense, my committee raised the question, "Why do you study women in STEM and why is this issue important to you?" I appreciated the question, which guided me to continue inquiring about GEMS as a space to empower girls and women. In this section, I explain why we encourage women to participate in STEM and the need for GEMS to provide an opportunity for girls to engage in STEM beginning at an early age.

Women in STEM

In college, Laura began to see that women did not have the same opportunities to take the science and engineering courses that men did. She recognized that these courses would lead to innovative fields, which would mean better paying careers and higher social status. She became angry over the gender injustice of having only a few girls enrolled in these classes, which meant that only a few girls would be presented with opportunities for good STEM jobs in the future.

For four decades now, gender-associated perceptions of the self remains prevailing as individuals making career decisions. I have worked with a team to develop and implement an integrated STEM project in a local high school STEM class. In the beginning, our team observed the STEM class to become acquainted with the students and later we implemented our project in the class. It was an engineering-focused class, with 11 students in total and only one girl. After the class, I had a chance to talk with her and she told me that she had taken this course twice because she enjoyed using equipment (e.g., 3D printer) to create new products. When asked about her career plan, she explained that she planned to be an English teacher. When I told this story to Laura, she said:

> I'm glad that this young woman wanted to be an English teacher, but why isn't someone igniting this passion for engineering and math? Because she's obviously very good at it. Why isn't somebody saying "You need to be . . .? Let's see if we can find something that will really excite you." You see what I mean. (Laura, Interview, October 2019)

Gender norms influence educational and professional decisions by impacting individuals' perceptions of accessible fields and self-identities in optional fields. If, at an early age, girls do not consider STEM fields and do not take courses in STEM, later their career options will be limited. Laura explained:

> What I don't like to have happen is for girls to limit themselves. [If] they don't take the hard math and science, then they're limited in college. And once they find something they really like, it may be too hard for them because they didn't have it in high school. That was why I originally started GEMS—it was to say to girls, "Take all the courses you can so that when you get to college, you have lots of choices." (Laura, Interview, June 2021)

STEM fields are associated with technology and scientific innovation, which lead to future job growth and individuals' economic and social security. Laura and I have discussed the importance of encouraging women to enter STEM fields many times. She is confident that doing so can ensure women's financial independence and broaden their career options, which contributes to social justice regarding gender equality. She said:

> First of all, if they end up in a STEM career, they're probably gonna make more money, which is important. When I talk to parents, I always say, "The days are gone when your daughter can get married and be supported by a loving husband her whole life. She's going to have to work. There's divorce, children, illnesses, they want to have a big house. I mean, she's gotta have a way to support herself. She needs a way to support herself. So, first of all, money, and it's not the most important thing. I also think GEMS does a good job of talking to girls; there's a lot out there that you don't know about. They may never hear some of these words like *chemistry* or *biomedicine* until somebody comes and talks to them about it and says, "I did that, I went to school and learned that, and it just opens a lot of windows." To girls who—maybe their parents never think of them other than as a nurse or a teacher, or their parents don't know any other things to expose them to. Their teachers may not really think that way. So, it's between exposure and more money. It just gives more options. And I want girls to have options. (Laura, interview, June 2021)

In addition to promoting women's participation in STEM education, increasing the retention of women in these fields also warrants attention. Women often do not feel welcome in male-dominated fields. Laura has been striving to improve girls' and women's involvement in STEM fields for 28 years. She often is frustrated by this reality:

> I know the statistics show that more and more women are going into these fields, but they're not staying. So clearly the issue is not fixed. I mean, they're just not staying in the field, so the climate is really bad. They may get a degree, but they can't stand to work there. So that's what makes me angry. (Laura, Interview, June 2021)

Laura insists that careers in STEM fields are beneficial for women. She acknowledges the dilemma that women who encouraged to pursue STEM fields may face more challenges than they would in traditional women's fields. She said:

> Part of me thinks why are we encouraging all these girls to go into STEM? When they get there, they'll be sexually harassed, or they'll be not able to be promoted. When I look back, I think I'm pushing these kids into this and they're certainly smart enough to do it. But when they get there, it's going to be much harder than if they decided to stay in a traditional field. Is it worth it? (Laura, Interview, June 2021)

Though sometimes she worries about pushing girls into STEM fields with potentially unfriendly environments, Laura envisions that increasing the representation of women in STEM would change the climate. She continued, "But on the other hand, if women don't go into them, it will never change. I do think the more we get in there, it has to change." GEMS provides a space for girls to experience STEM activities and allows girls to see career paths in STEM fields.

GEMS as a Girls-only Learning Environment

In a journal entry, Laura described her experience in explaining gender equity in the first GEMS club. She wrote:

> Up to this point, I had been careful to avoid an agenda of gender equity. I just wanted to present a set of activities in a safe environment and see what happened. So, the girls had not been hearing anything about boys and girls in the classroom. . .. I also was worried about the political nature of this agenda. Gender equity is a very sensitive subject. Many people do not feel that there is a problem, or that any particular group should have special treatment. If I did this again, I would ask parents' opinions about the quality of the project and not be so concerned that their opinions might derail what we were trying to do. (Jones, 2002b, p. 4)

At that time, Laura did not want to bring the issue of gender equity into GEMS. Instead, she explained to girls, parents, and schools that girls deserve extra time, attention, and encouragement in mathematics and science.

More recently when I met Laura, she was comfortable talking about gender equity and women in STEM. She responded to people who were concerned about GEMS excluding boys by arguing:

> Do you have a robotics team? How many girls are there? Do you have a computer science class? How many girls are in there? . . .. Look at the statistics and see what's happening. Because boys have many opportunities to do these things that don't necessarily exclude girls but don't encourage girls. We're providing a place to encourage girls and we're not saying they should be in girl-only things the rest of their lives. But let them have a really nice positive experience. (Laura, Interview, June 2021)

In the GEMS Handbook, Laura explicitly explained why we need GEMS. She wrote:

> Why not just tell girls to do more science? Why not just buy girls chemistry sets and LEGO kits? Why not just tell parents to encourage their daughters to take more math courses? Our experiences as GEMS club leaders and mothers show us that it just doesn't work that way. Girls need more than one invitation, more than one encouraging voice. And they like to do things with their friends. They also need to feel that they can try new things without risk, without fear of "breaking it." And that is exactly what a GEMS club does for a girl—it gives her the chance to try new things with her friends and to succeed without the pressure of grades, test scores, or boys watching. (GEMS Handbook, 2019, p. 9)

GEMS is a place where girls are the center of attention. Laura claims that girls need such a space to become agents of learning and make their own decisions. The environment in GEMS provides an opportunity for girls to practice being themselves and make decisions without seeking confirmation from other people. Laura argues girls need a space like GEMS to validate their voices and experiences in order to build confidence. Laura has built GEMS culture through collaboration, friendship, excitement, encouragement, and challenges. In the GEMS Handbook, Laura addresses the leaders:

> You will find that your GEMS club gives many girls their first experience of using tools, mixing chemicals, wearing goggles, making mistakes, and laughing. These are wonderful experiences for every child, and they are the making of scientists and engineers. You will also find that your GEMS club gives you experiences that you will never regret—the excitement of young girls understanding a difficult concept, the laughter as they experiment and explore the materials, and the eagerness with which they greet you each meeting. Girls are hungry for these kinds of experiences, and you are the lucky one who gets to provide them and share their enjoyment. You are changing lives! (GEMS Handbook, 2019, p. 9)

Specifically, Laura often emphasizes that GEMS leaders should be females. Some people do not understand and question this rule rejecting male educators. Laura wrote in her journal:

> If someone asked me what I want in a co-leader, I would say a woman (of course) who thinks that girls are important, that girls deserve extra time and attention, that girls need and deserve encouragement in math and science, and that extra time and participation in math and science in a fun way make a difference. (Jones, 2002b, p. 4)

Recently I discussed this with Laura. She replied, "If you can't find in your school a female teacher who is interested in math and science, that should be a problem." Girls need female role models in GEMS. Though the leaders are not necessarily experts in STEM fields, they bring activities into GEMS, which is a way to communicate with girls that women and girls can do STEM. In the GEMS Handbook, Laura also encourages parents to use role models to inspire girls in daily life:

> Find role models in everyday life. If given the opportunity to choose a new doctor, choose a woman. Ask the female pharmacist (in front of your daughter) about her education. Ask the neighbor who is a computer engineer to tell your daughter about her job, or to take her to work for a day. (GEMS Handbook, 2019, p. 20)

GEMS not only provides learning opportunities for girls, as discussed before, but it also creates a culture where girls and women can construct their own knowledge.

In the recent study with the OGGs, I learned that the girls-only feature of GEMS had later impacted their parenting. One OGG, Kate, wrote in the survey:

> For me, it was always figuring out how to show that I was as good as the boys and I will want my daughter to know the same. I will definitely be exposing her to all types of math, technology, and science as she grows up and to know that she can do anything. (Kate, GEMS Original Girls Survey, 2020)

Another OGG, Millie, reported:

> That alone, to me, has a tremendous impact on how I will seek opportunities for my children. To find things that are engaging like GEMS and provide alternative ways to the classroom for learning since different people thrive and learn differently. I personally love this way of learning (through activity) the most. (Millie, GEMS Original Girls Survey, 2020)

In GEMS, learning and teaching are based on care and interpersonal communication. The girls-only environment relieves the tensions of gender stereotypes in learning mathematics and science. Thus, GEMS knowledge builds on trusting subjective experiences, feelings, and intuitions among girls and female leaders, while competition is less focused within the group.

Chapter Summary

In this chapter, I focused on GEMS as an ongoing educational community which provides an alternative learning opportunity for girls in mathematics/STEM. Drawing from a variety of sources that included Laura's journals, interviews with Laura, GEMS documents, interviews with GEMS leaders and GEMS girls, GEMS meeting notes, and research memos, I have explored the history of GEMS, GEMS leaders, and GEMS as a learning environment that provides a girls-only environment to promote girls' participation in STEM fields. In this chapter, In the next chapter, I focus on our mathematics experiences and evolving views of mathematics. Through exploring our experiences with GEMS, I describe the role of mathematics in GEMS. I also share the two emerging mathematics tasks in our collaboration.

CHAPTER 6. MATHEMATICS ACROSS BOUNDARIES

Mathematics is an important theme in this narrative inquiry study. When I learned that Laura started the first GEMS club because her daughter complained that "math is hard," I began this inquiry by wondering how mathematics is presented in GEMS. As a mathematics teacher educator, I bring my understanding of mathematics into our communication and GEMS work. In this chapter, I first explore our mathematics experiences and evolving views of mathematics to illuminate our conceptualizations of mathematics. Then, through exploring Laura's experiences with GEMS and my own experience in leading a GEMS club, I describe mathematics in GEMS in the past, present, and future to understand the role of mathematics in GEMS. Finally, utilizing a boundary crossing lens, I describe emerging mathematics in our collaboration.

Conceptualization of Mathematics

Mathematics has a unique status in school and real life in that it is an essential component of school education and is taught as a compulsory subject throughout K-12 education. On the other hand, mathematics produces anxiety and failure for many students. Even those who have successfully completed mathematics courses often do not think mathematics is easy to learn. People often experience extreme emotions related to mathematics, such as hate or love; that is, very few people have neutral feelings about mathematics. We all have mathematics stories to tell, and these stories pervade our understanding of who we are and what mathematics is (Hottinger, 2016).

Looking Backward: Mathematics Experiences

Laura: I Was Not Encouraged to Use My Strengths

As I described in Chapter 4, Laura was a high-achieving mathematics student. From fifth to eighth grade, she was enrolled in a gifted class. Laura does not have particular memories about mathematics at that time. In an interview she shared: "I don't have any memory of doing math or science in elementary school, and I know we did. I remember it was not a struggle. I never thought it was hard. So, it must have been fine." Laura also could not remember what mathematics courses she took in high school. In a scrapbook her mother made for her, she found high school report cards and brought them to the interview. She showed me the courses she took and grades, and said:

> I found my report cards; I was in the honors classes, and I took Algebra, Geometry, and Algebra II, and that's all. They had Trig, they had Calculus, they had AP math. I never took them. You see, and it's like—what a waste. (Laura, Interview, August 2019)

She further lamented:

> Here's the sad thing. You think back and you think a guidance counselor who sees As in honors classes should have said to a girl student, "You need to take higher-level math." Never a mention of it. It was really—and that was the way it was back then. Now, I'm sure somebody would have sat down with any girl who is getting As in these honors classes and say "You're not stopping at this math." (Laura, Interview, August 2019)

Laura copied her high school report cards and handed them to me and said, "You can keep them. I'm happy to share the grades and the scores, because they made me feel good, but they also made me realize that I had definitely not used my strengths."

In college, Laura took three mathematics-related courses for elementary education. She told the story about the first mathematics course which she did not actually take but got an A; she said:

> When I came to college, I tested out of a bunch of stuff, and that's another reason I could get through college [in three years]. I walked into that one [mathematics course], and the instructor gave us a pretest. This is funny, and I got 100%. He said, "You don't need to come back. I'm giving you an A." That was cool. (Laura, Interview, August 2019)

Even though Laura achieved high marks in mathematics, she did not see that her abilities were developed in the direction she should go. She sighed, "Nobody said, 'You're really good at it [mathematics]; you need to go into math and science.'" She further shared, "I lost my opportunity." Her regrets ultimately motivated her to establish GEMS, noting, "I don't want girls to lose their opportunity like me."

Lili: Math Is Infused in My Life

In Chapter 1, I recounted my mathematical stories. Exploring Laura's experiences with mathematics afforded me an opportunity to reflect on the importance of mathematics in my life. Mathematics has been a key for me to access higher education and to go through social transformation. Although learning mathematics was not always going smoothly for me, I often could perform well on tests. In high-stakes examinations, my aptitude for mathematics granted me access to college and graduate school.

I was also recognized by teachers and peers as a girl who has a "mathematics brain," which provided the inspiration for me to pursue mathematics. My high achievement in mathematics contributed to the development of my mathematical identity. An awareness of both my mathematical achievement and identity confirmed my decision to study mathematics. I am grateful for the recognition I received that opened a door for me to progress further in education and navigate a career path. However, like Laura, who was not given much advice in high school, I also did not receive suggestions from teachers on career directions related to mathematics. The school and teachers cared much more about the college enrollment rate, in particular, a high-ranking college enrollment rate, which is a standard by which to evaluate the quality of a school. Thus, my teachers were concerned about which university I could attend, rather than which field I would pursue. Looking back, I never considered other fields as options when I made my career decision.

In the Midst of Mathematics

Geometry and Algebra

Laura remembers that she liked mathematics in high school. She said, "I know that I liked math. I did math in my head like little games, and I liked doing codes and I liked playing with that stuff." Laura explained that she has a great interest in algebraic and logical problems:

> I remember doing things like ciphers. I'd check a book out of the library, and I'd play with that. It was just something I did that I don't think anybody knew about. It was just kind of like a little fun thing to do to practice analyzing it in my head, very interesting. (Laura, Interview, August 2019)

This was her little secret and she never told people; she did not want other people to know that she enjoyed doing strategy games, like ciphers. For Laura, doing this type of mathematics was not quite related to learning in school but it gave her joy. Laura explained, "It was always kind of algebraic stuff. It was never geometry." She expressed her strong negative perception of geometry: "I hated it. It was like, why [do] you have to prove something when it's very clear that that's a 30-degree angle?" Laura was surprised when she reviewed her report cards that she did not fail Geometry. She reiterated, "I hated geometry. I kept thinking what grades I got in Geometry. I got

As in Geometry all the way through, and I hated every bit of it. So clearly, I'm not as stupid as I thought." For Laura, achievement does not align with her identity in mathematics.

Laura supposed that she does not like geometry because of poor spatial skills. She reflected:

> I think that I didn't really understand the spatial part of geometry, and so to me they—algebra and geometry were two totally different things. It may have been the way they taught it, but I hate this geometry. I'm clueless about spatial stuff. I mean, I was a child who should have played with LEGOs. (Laura, Interview, August 2019)

I heard Laura's views about algebra and geometry and reflected on my experiences with these two areas of mathematics. In a reflection, I wrote:

> Since middle school, I always had the same mathematics teachers who taught both Algebra and Geometry. When I was a classroom teacher, I also taught both courses. I do recognize they are different mathematics. But I understand them in more connected and integrated ways. I think because Chinese mathematics teaching emphasizes a mathematical thinking called combination of number and shape 数形结合. Literally, the combination of number and shape is to connect algebraic and geometric representations. The more we practice this mathematical thinking in learning and teaching, the more natural we see in algebra and geometry. I see multiple representations are emphasized in the U.S. curriculum, but [they do] not explicitly connect geometry and algebra. (Lili, Research Memo, October 2019)

Later, Laura learned that girls' insufficient experiences in spatial practices hinder their growth in spatial reasoning, which then limits their development in STEM fields. This confirmed her own struggles in geometry: "It's still a major weakness now. I think this whole thing was girls and spatial skills."

Memorization Versus Understanding

Laura and I had conversations about how to learn mathematics. Laura told me her concerns about current education that emphasizes conceptual understanding and focuses less on memorization. She is worried that the promoted ways of teaching mathematics hardly ensure effectiveness. She stated:

> That's just the impression I've gotten from, you know, people who talk about how they teach math now, but I can't imagine trying to function without knowing addition and multiplication and all those facts. I mean, how can you do math in your head without being able to quickly do that? And formulas, I mean, even when I'm making a quilt, I'm thinking about, "Oh, yeah, that's how you find the distance of the hypotenuse," so I know how much fabric I need to cut it into a triangle. I don't see a lot of problems with the way we learned math when I was—back in

> when there were dinosaurs. I guess there are reasons why they're doing it now. But I think a lot of stuff in math needs to be memorized so that you can use it. (Laura, Interview, August 2019)

The conversation reminded me of my father's viewpoint on mathematics learning. He believes some concepts are impossible to understand for younger students, but they still could follow some patterns or memorization to practice mathematics. My father and I debated many times about whether a child should follow the mathematical procedures without understanding the concepts. My father's point is that conceptual understanding can be developed during repetition and procedure practice; if teaching waits until students fully understand concepts, they will lose many learning and development opportunities. I do agree that teaching should not be delayed but I believe teaching needs to support students' conceptual understanding rather than providing formulas and procedures to memorize. In my early school years, mixed operations confused me, and my father would characterize me as careless for making mistakes rather than taking poor understanding of concepts into account. He mainly focused on the correctness of procedures and results, rather than the process. When I reached the upper grades and understood why the procedures operated in such ways, I began to develop an interest in mathematics. My father insists that the earlier practice helped my later understanding.

I do not believe memorization can help understanding, but I do agree with Laura that some memorization is necessary and does not conflict with mathematical understanding. For instance, recalling multiplication tables can help people solve many real-life problems. But people need to understand multiplication; otherwise, they will not recognize the context in which they need to use the multiplication tables. Memorizing the Pythagorean theorem can help students quickly solve problems related to right triangles, in particular, during tests with time constraints. Students do not have to prove it every time they use the theorem, but they need to understand and be convinced by the proof. Then when they use the theorem, they can trust and justify their strategies.

I understand Laura's concerns. Through my daughter's mathematics homework and her strategies, I have indirectly observed her teacher's instructional practices. In mathematics curriculum, manipulation or visualization is intended to provide multiple representations to increase students' conceptual understandings (e.g., CCSSI, 2010; NCTM, 2000). However, multiple representations often become formalistic in teachers' practice, neglecting individuals' needs. For students who have already developed abstract thinking, the use of visualization might not further their development in mathematics. For students who benefit from concrete examples, they need solid support to move from specialization to generalization.

Looking Forward: Promoting Mathematics

Spatial Reasoning

Laura recognizes that spatial reasoning not only contributes to geometry learning but is also important in real life. She enjoys making quilts, which includes knowing how much fabric to use and how to cut the fabric. Laura explained:

> That is an example of my bad spatial skills, because I have no clue how much fabric I need for anything. That's a really practical application of angles, because, you know, I'm sitting there looking at this border and I was gonna estimate how much I need. I don't know. I don't want to do the math all by four yards. . .. That's the kind of thing that in real life spatial skills are really important. Somebody will say, "Well, it's just 500 yards down the street." I have no idea how much that is—three miles? No idea. (Laura, Interview, June 2021)

Laura shared her experience with spatial problems, but she does not believe spatial ability is fixed. She pointed out that people often believe that having spatial skills is an innate talent:

> I think most people would say, "I have a good spatial sense," or "I don't." They don't see it as something they can learn. So, when kids are little, they're not paying attention to them [spatial activities]. And the kids either get the experiences or they don't. If they don't, then they're starting off, later in life, at a big handicap. Girls, kids who haven't played with boxes and that kind of 3D models—they just don't get these experiences. (Laura, Interview, June 2021)

One time, Laura and I discussed the connections between spatial skills and geometry. I thought spatial reasoning should be developed in geometry because it is a geometric skill. Laura stated that spatial skills are related to geometry but often are not included in the school geometry curriculum. When Laura and I were updating the GEMS website, she suggested adding an independent webpage about spatial skills rather than under other GEMS webpage (e.g., mathematics): Specifically, Laura contended that school curriculum does not provide sufficient learning opportunities for students to develop spatial skills. She stated:

> I want to do more. I see girls struggle in spatial reasoning. I see it every time when we do something with the kids. If I could give every girl a box of LEGOs. It's not just LEGOs. That's not taught in school. It all has to be taught informally, just learn those spatial skills. We know that this holds girls and underserved minorities back.

> We know the [school] education system does not do enough. If GEMS can do something about that, then at least we can get a few girls to have a little bit of experience, so it's not so frustrating. I don't want anything that we know will hold them back to hold them back. (Laura, Interview, October 2019)

Laura is concerned that a lack of spatial skills can prevent girls from pursuing mathematics and other STEM opportunities. Laura seeks every opportunity to reach out to people and ask for suggestions regarding spatial curriculum. She said, "Having a spatial curriculum is the only thing I want to do for girls before I die."

Expanding the View of Mathematics

Although Laura had excellent grades in mathematics, in our collaboration, she always claims, "I am not good at math." Laura said that mathematics was "too school" to bring into GEMS, because it is hard to do mathematics activities without focusing on right or wrong answers. Laura admitted she had not brought many mathematics activities to GEMS. However, I told Laura that I could identify a mathematics component in nearly all GEMS activities. I thus asserted that mathematics is everywhere. Laura laughed:

> I guess I need a field trip with someone who takes me to see math everywhere. I see technology, engineering everywhere. I don't see math. I do see math at the grocery store. I see it when I'm out shopping. (Laura, Interview, June 2021)

Laura acknowledges that her view of mathematics is primarily limited to formal mathematics. In real life, she often does not recognize or acknowledge mathematics. Laura reflected on her views of mathematics in her collaboration with the Purdue GEMS team, consisting of mathematics education faculty and graduate students:

> Listening to your group, and everybody talking about math—It's been really good for me. It's opened my eyes to a whole lot of research and a whole lot of [mathematics] concepts and stuff that I'd never thought about. So, it's been wonderful. There are so many times when you guys say, well, that's math. And I'm sitting here thinking, "I don't see it. Help me out here." But because everybody else nods their head and they know, I don't want to look like a fool. So, I guess I need to be more assertive. It's "Tell me what you're talking about." I think people, you guys, all kind of have a similar focus about what math is, which is great. But I don't completely understand it. And there's nothing wrong with it. I just, it's way wider than I understand. And that's good. There is nothing bad about this collaboration. This is so good for GEMS. (Laura, Interview, June 2021)

Laura sees us as "math people" who have authority in mathematics. Further, Laura has considered promoting mathematics in GEMS by broadening views of mathematics: "We need to make GEMS leaders and girls aware of the wider range of math. And that I can't do that myself; you guys are the experts." During the collaboration, Laura began to rethink mathematics by saying, "In general, my view [of math] is different; my view is much narrow[er], but I've loved expanding it." Laura's unwillingness to ask questions for fear of making a mistake reminds me that people view mathematics as being about right or wrong answers and that people who do mathematics are special and isolated from ordinary people. Many people, even those like Laura who has taught mathematics activities in informal spaces for more than two decades, still reluctantly explicate mathematics concepts and mathematical ideas in the activities and do not feel comfortable talking about mathematics.

Mathematics in GEMS

GEMS was started with the goal of promoting the enrollment of girls in advanced mathematics and science courses. In this section, I continue the inquiry by exploring the role of mathematics in GEMS and how mathematics is promoted in GEMS. I begin with Laura's description of mathematics in GEMS, which reveals her evolving views of mathematics. I then relate my own experiences with mathematics in my GEMS club, which informs a fluid identity in mathematics. Finally, I share our discussion about crossing disciplinary boundaries to promote mathematics in future GEMS clubs.

Mathematics Was Dropped Out

Laura recognizes that mathematics in GEMS presents a dilemma; on the one hand, the goal of GEMS is to increase girls' interest in mathematics; on the other hand, Laura has discovered that engaging girls in mathematics can create tension with another goal, presenting GEMS as different from school. Unlike other subjects, such as engineering or science, that are easily integrated into hands-on activities and experiments, Laura sees mathematics as built on mental strategies and memorization, which could cause girls to lose interest very quickly. Laura recalled her experiences with the first GEMS club:

> It was 1994, so the internet sure wasn't available—and so I would go to the library, and I would get books out, and I'd bring that something and say, "Can we try this?"

> It was a joy thing of just fun things to try, but it was interesting. I remember we would try to add math things, and neither one of us [Laura and Ms. Cooper] did it very well. And we realized quickly that we needed to be careful—we weren't good at the math part, so we kind of dropped out math, because it was so much like school. And so that's why I'm so interested in how we're going to get math to be more engaging. (Laura, Interview, August 2019)

Laura has struggled with the certainty of mathematics that may diminish the passion for exploration. She has expressed concern that, as GEMS girls seek a correct answer, they might overlook the opportunity to discover other possibilities in the process. If a problem is challenging, it can be perceived as difficult and frustrating. Students who are not confident in mathematics thus fear failure and may lose interest and the confidence to explore. Laura said:

> I think with math—what was always so hard for me is that it's very tricky to make it not be the right answer—you see, to experiment. I don't even know how to begin. With the technology, when we would do computer stuff with kids, it's something that they could do at home. It's not just school stuff. It's something fun, and so that's what I mean—not like school. It's not intended to put math down. (Laura, Interview, August 2019)

Laura assumes that many other GEMS leaders have the same issue with mathematics:

> Well, I'll be honest with you, because I've always said to the teachers, do what's fun for you. They're not picking math. I will guarantee you they're not picking math, because they might say it's a tool. Now, I will say, last year at a middle school, one of the leaders was a math teacher, and she did something with pumpkins. It was something related to math, and it was probably pretty concrete. I don't know if they were figuring out the circumference by playing around with rolling them or something. I know she did something related to pumpkins and math, and the kids liked it, but she's a math teacher. I think anybody who's not a math teacher and doesn't love it, they're just going to let [mathematics] go. (Laura, Interview, August 2019)

Even when mathematics is present, leaders may not acknowledge it is mathematics. Laura suggested carefully choosing language to communicate with girls about mathematics:

> I do think if there is math in there, like the tangrams or something, as GEMS leaders, they may just see it as a puzzle. You could say to girls, "You may not realize, but when you are building those, and you're manipulating those shapes, you're using [math]." You could almost say "You're using the kind of math skills that an architect could use." (Laura, Interview, June 2021)

On the GEMS website, we provide sample activities for leaders to use. Some activities are labeled as science (e.g., Extracting DNA) (GEMS Science, 2022) or engineering (e.g., Engineering Magic) (GEMS Engineering, 2022) on the website, and mathematics components can be identified in the activities. However, mathematics is not emphasized in the activities. I expressed my concern that mathematics is often hidden in GEMS activities. Laura proposes that leaders do not label activities at the beginning but later guide students to reflect on the subject knowledge. She said:

> There's a lot of math in there. I think we're going to have to help the leaders, not label it or maybe make it a mystery. At the end after you do these really fun activities—let's think about what kinds of categories we were using today. Were we using science or [math]? The teachers are really going to have to rethink how they bring math into GEMS. (Laura, Interview, August 2019)

After noticing Laura's hesitancy to communicate directly with GEMS girls about mathematics, I asked if she was concerned that by emphasizing mathematics, girls would lose interest in GEMS. Laura answered:

> No, because again, as long as we always have to keep at the front that it's not school and it's fun, I don't think there's going to be a problem. Because there's going to be hands-on activities. I'm not worried about that. I think as long as we keep those things at the top. (Laura, Interview, August 2019)

Presenting mathematics in a different way from school—fun and hands-on—is Laura's goal for promoting mathematics in GEMS. In collaborating with the Purdue GEMS team, Laura expects we can continue to offer mathematics activities and even highlight the math and said, "I'm really interested in what you guys come up with for fun math in GEMS, because this could make a huge difference."

Where Is the Mathematics?

In the summer of 2021, I led a GEMS club in my neighborhood. Four to seven elementary girls in the community, including my daughter, joined for each meeting. We met twice a week, for a total of 10 meetings. I provided supplies for the girls based on the activities I had modified or designed. I did not label the activities by subject, such as a math activity or science activity; instead, I called each activity an experiment or game.

I intentionally designed activity questions to prompt the girls' mathematical thinking. In an activity called *Solving a Problem in 50 Meters*, I developed a set of mathematics problems and integrated them in a game. The girls were asked to run a 50-meter dash. On each line, at the halfway point, I put a piece of paper on the ground with a mathematics problem. The girls needed to stop at the halfway point, pick up the paper, and solve the question, then ran the second half of 50 meters. At the end point, they came together, shared the questions they had, talked about their thoughts on the problems, and discussed others' problems. I tried to write the mathematics problems in an interesting way. For instance, I asked the girls to write three numbers that are bigger when you read them from left to right than from right to left, or to draw a shape inside another shape and name the two shapes. The girls loved these questions; they talked, discussed, argued, and laughed.

In another activity, called *Floating Egg* (see Appendix B), I guided the girls to conduct a scientific experiment. The principle of the experiment is to add salt to water to increase the density of the salt water, which makes an egg float in the water. In the activity, I asked the girls to record the amount of water and salt, and the distance of the egg in the cup from the table. After the experiment, I invited the girls to share their thoughts on the phenomena and principles of the experiment, based on their observations and the data they recorded. I facilitated a conversation among the girls, thinking about the mathematical relationships among the variables. At the end of the discussion, one girl asked me, "Ms. Lili, where is math in this activity?"

Where is math? I remember I was stuck for a moment. In the reflection, I wrote:

> The girl asked me where math is. I knew I needed to carefully organize my words to communicate my understanding of mathematics with a third grader. I did not answer her directly but asked her, "What do you think math is?" She told me math is about numbers; you add, subtract, or multiply. Since we did not do these in the activity, she did not know whether we did math. Then I shifted the focus from math to the activity and asked her, "If I add one tablespoon of salt to four cups of water, can the egg float?" She looked at her recorded data and thought for a while. Then she told me she didn't think so. I asked, "Why?" She pointed to her data and said, "I added five tablespoons of salt to two cups of water; the egg only goes up a bit. So, I think one tablespoon of salt and three cups of water can't make the egg float." I told her I was convinced by her [argument] because she drew information from the data to make a reasonable conclusion. I said to all the girls, "That is math. See, you are doing math." The girl said, "Oh, I thought that was not math." (Lili, Reflection, June 2021)

In my GEMS club, mathematics was highlighted in a fun way. Girls were engaged in and enjoyed the integrated activities. By the end of each session, they asked what the next GEMS activity would be. Leading a GEMS club has informed my views of mathematics—that I see mathematics is broader and the disciplinary boundaries are blurring.

Cross-Disciplinary Boundaries

Looking forward, in our desire to highlight mathematics in GEMS, Laura and I have thought about encouraging leaders to communicate about mathematics with girls. Laura stresses the role of mathematics in career paths and suggests an indirect way to draw attention to the mathematics:

> Maybe we should encourage leaders at the end [of the activity] to say that, in addition, as a career, you could use the skills. You could also say, "And by the way, you may not have realized, but when we were building those LEGOs, we were using math words." And then what you hope will happen is the girls walk away thinking, "Oh, today I had a fun time doing math words." Or, "I had fun today when I built it, because I was doing some science and I was doing some math," and it's just kind of like bringing the awareness of what you did to the foreground. Even if there isn't a whole lot of it there. Pointing out what you're doing makes a difference in how people perceive it. (Laura, Interview, June 2021)

From Laura's viewpoint, changing the language to point out the mathematics but not position it as the center of attention provides an opportunity for the girls to see mathematics as not so frightening. As the mathematics continues to be exposed, the girls will become accustomed to working with it; in this way, they will begin to build confidence in their mathematical abilities.

Rather than treating mathematics as an individual subject, Laura also suggests that revealing the mathematics in integrated STEM activities would help the girls see mathematics in a broader way that connects with real life and other STEM disciplines. As such, the girls would view mathematics not just as a school subject but as normal practice. Laura realizes that the structure of the GEMS website should be more integrated. She said:

> We may have gone about this the wrong way, and that we have it all separated out on the website. We have science, we have math, we have engineering. That's not really true STEM. And so, we're gonna have to figure out how to, to describe it in a more integrated way. (Laura, Interview, June 2021)

Integrated STEM goes beyond the disciplinary boundaries to allow leaders and girls to build connections among disciplines. Laura suggests that spatial reasoning itself is an integrated STEM area, as we build connections among individual STEM disciplines through spatial reasoning. Laura explained:

> I think when we bring in the whole spatial skills thing, and when we look at things like topology, and other forms, like computer network design. What's behind stuff like that? It kind of naturally integrates itself. (Laura, Interview, June 2021)

Laura continues to collect information about spatial reasoning and has gathered insights from experts in Geographic Information Systems (GIS). Developing spatial activities is important work for the Purdue GEMS team. Laura and I have taken an online course and have met with experts to learn about GIS and how to incorporate these ideas into GEMS activities.

These new experiences have informed my teaching knowledge. I encourage my pre-service teachers to think about mathematics in connection with other subjects when they plan their lessons. For instance, one pre-service teacher who wanted to implement a STEM activity in her field classroom asked me for suggestions. I invited her to attend the GEMS meeting where we would help her brainstorm some ideas. During the meeting, through a GEMS activity called Hidden Building (GEMS Spatial Skills and Reasoning, 2022), we discussed the mathematical ideas in the activity and the possibility of connecting with engineering and technology. The next day, Laura said:

> I thought it was interesting yesterday with [the pre-service teacher]. I think we opened her eyes just a little bit to the wider range of math, and how it could be combined with fun stuff. But I just think that the conversation was so interesting about building on what looked like an engineering activity. Do you know what I mean? Building it into a spatial skill on a math activity, and I think it opened her eyes a little. (Laura, Interview, June 2021)

By promoting mathematics in integrated STEM activities, Laura claims that "math continues to be pointed out, forefront, praised, and improved."

Evolving Mathematics in Collaboration

Laura describes GEMS as an afterschool program that includes an intervention to provide alternative learning from the classroom for girls. In particular, Laura often emphasizes that GEMS is different from school in that it opens the way for learning possibilities from emerging opportunities. Laura does not want the leaders to feel required to use a specific set of activities, saying, "I don't ever want to say to [leaders], 'You have to do these activities,' because that takes the joy out of it." By not following a specific curriculum, leaders have more freedom and autonomy to be curriculum developers. In this section, I share evolving mathematics in our collaboration. I describe two mathematics tasks to illustrate informal educators as curriculum developers who capture emerging learning opportunities from everyday practice and teaching practice. Specifically, I describe the development of the two tasks through a boundary crossing lens, that is, from identification, coordination, reflection, and transformation—four stages to describe our experiences with the two tasks.

Quilt Task

Laura likes sewing; in particular, making quilts is a fulfilling part of her life. She sees making quilts as a way to create something and make life meaningful. I do not have experience in making quilts, but I enjoy designing and making clothes by hand-sewing. As part of our ongoing relationship, we continue to share our creations with each other. The quilt task I describe in this section emerged from our daily communication. Specifically, using boundary crossing theory (Akkerman & Bruining, 2016), I articulate the four stages of the task development: identification, coordination, reflection, and transformation.

Identification

One day, Laura described a problem she had encountered in making a quilt: "It is about enlarging a quilt and I am so, so bad at spatial stuff." A little later, Laura sent an email titled, "I need help," along with a math problem:

> Here is the quilt pattern. I am trying to figure out how to make it a little bigger because I have gorgeous fabric, and I want to expand the portions marked 28 inches wide to about 38-40 inches wide. Unfortunately, they will stay the same length. The problem I am having is many-faceted:
> 1. Do I make the square bigger? If so, what size?
> 2. Do I change the location of the angled cut to make it more proportional?
> 3. What will the finished size be? (Laura, Email, April 2021)

In the email, she attached the pattern, a part of the pattern, as seen in Figure 1.

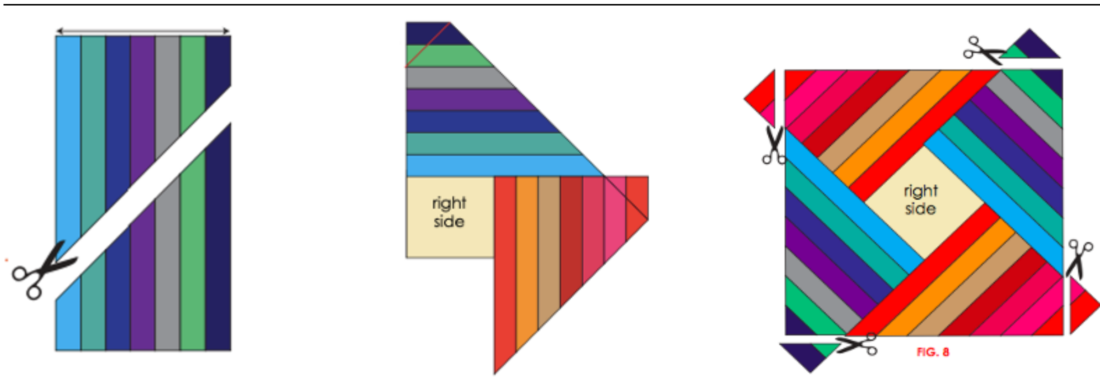


Figure 1. Quilt Pattern

When I read her email, it took me quite some time to make sense of the context. After I understood the pattern and Laura's request, I began to see it as a geometry problem about relationships among angles, sides, and areas. I grabbed a piece of paper to create a model of the pattern (see Figure 2) and responded to Laura:

> If you have 40x40 fabric, you can decide the size of the quilt. It also depends on how big of a square in the center you want. See the pic below. You can move the triangles to make the square inside bigger; then, the quilt will be bigger too. Theoretically, you only need to make four right isosceles triangles and the legs are 40. Indeed, you do not have to make triangles because one angle would be cut as I drew in the pic. When you decide the size, I can help to calculate the exact sides. (Lili, Email, April 2021)

Laura replied, "We need to talk about this. I swear I don't get it."

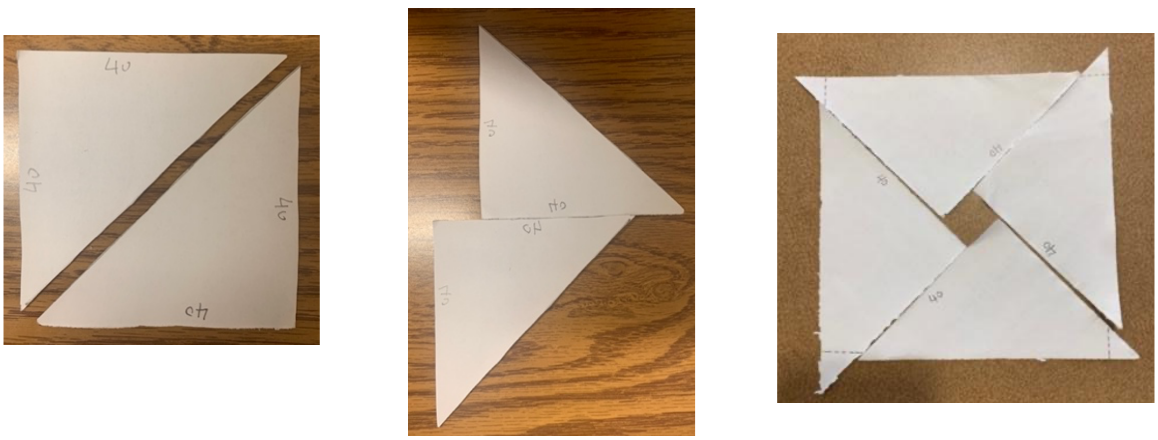


Figure 2. Paper Model of Quilt Pattern

Coordination

Then we set up a virtual meeting to discuss the model. I manipulated the paper to show Laura how to position the triangles. She told me she already made the center square, which is 12x12, and asked how to calculate the size of the final quilt. I told her the quilt should be 48x48. Laura said, "It is not large; I thought it should be bigger." As I heard Laura's concern, I realized I had made mistakes. In the reflection, I wrote:

> I immediately realized I used 40 as a hypotenuse, which indeed is a leg of the right triangle. It should be $40\sqrt{2}$. I told Laura I made a mistake—it should be 56. Laura laughed, 'Yay, you made a mistake! You, math person made a mistake!' We both laughed. (Lili, Reflection, April 2021)

Later, using geometry software, I made a model to illustrate this task and demonstrate the geometric relationship (see https://www.geogebra.org/geometry/wfgmts6s). The dynamic model uses different colors to show the way of cutting the raw fabric and the final quilt. Figure 3 is a screenshot of a moment of the model. Laura thought the visual model was very helpful for her to understand the spatial relations.

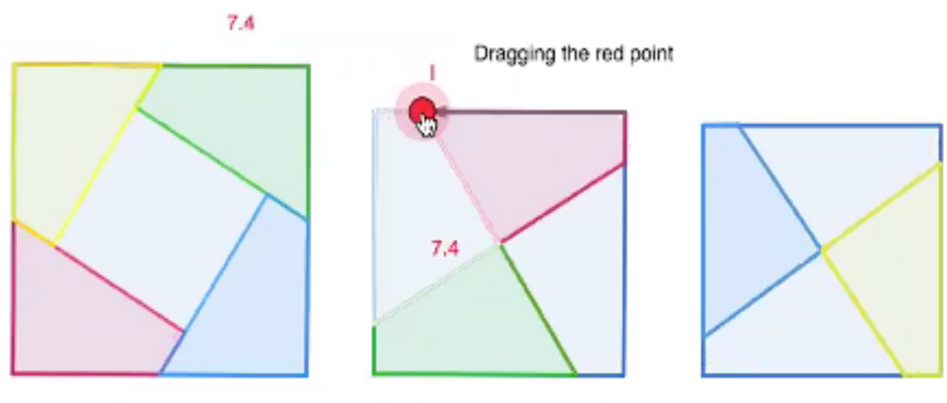


Figure 3. GeoGebra Model of Quilt Task

Laura and I recognized mathematics from this real-life problem and felt it would be interesting to turn this into a mathematics task. We worked together to develop a problem called *Ms. Jones's Quilt Problem* (see Figure 4).

Ms. Jones has a question. She is making a quilt for her sister. The pattern she used for the quilt is below. She has two fabulous squares of fabrics 40″ × 40 ″. She wants the finished quilt to be a square with sides between 46″–56 ″. Could you help her figure out where the cuts are and how you make the quilt?

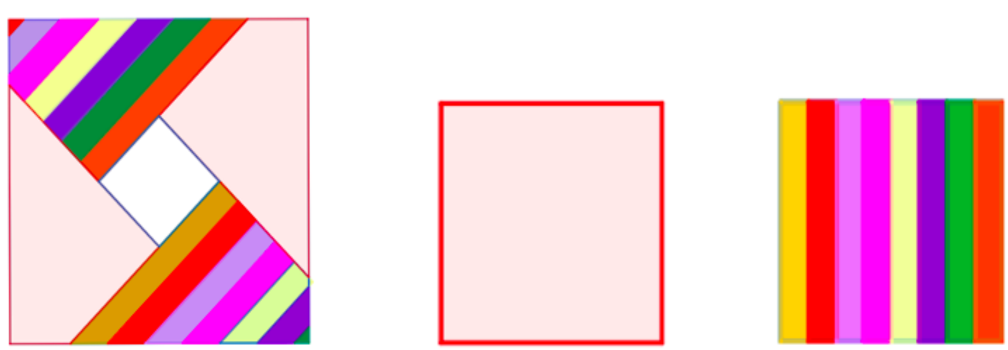


Figure 4. Ms. Jones's Quilt Problem

I brought this task to my pre-service teachers and asked them to try it and provide feedback. My pre-service teachers pointed out that more information is needed so the students have more to build from, such as information about quilting. They also suggested breaking the task down into smaller questions to help guide students. Since not all students know about quilt making, providing physical materials could help students manipulate and understand the task.

Reflection

Building on the feedback I collected from pre-service teachers and mathematics education colleagues, Laura and I both reflected on this task. Laura said:

> I'm clueless about spatial stuff. That is an example of my bad spatial skills. I also see my daughters' and other girls' struggles in spatial skills. They need to understand [spatial]; if they cannot figure out how to do it, the boys take it over. So, the only thing I want to do for GEMS is develop a spatial curriculum. (Laura, Interview, June 2021)

I reflected on the task and wrote:

> This problem is from real life and developed as a contextualized mathematics problem. However, as mathematical relationships are recognized, the context is not important anymore and is even removed in the solution process. It changes back to a pure math problem. We need to think about making mathematics tangible and engaging throughout the problem-solving process. (Lili, Research Memo, May 2021)

Laura and I were not very satisfied with the problem in the reflection. We both thought it was too much like school and not very fun to work with. Although the task is about spatial reasoning, it reflects the idea of optimization. This is not the mathematics that we wanted to communicate with the girls.

Transformation

Building on our reflections, Laura and I considered changing the task from solving a problem to enjoying exploration. We decided to expand the task to a project. Laura thought that making a real quilt was not feasible, so we agreed to use paper instead of fabric. We broke the project into multiple sessions and integrated art, culture, real-life elements, geometry, technology, and algebra into the sessions (see Table 2).

Table 2. Quilt Project

Session	1	2	3	4	5	6
Topic	Knowing patterns	Appreciate quilt designs and share stories about quilts	Use GeoGebra to draw your quilt design	List materials you need	Make your quilt	Exhibition
Element	Art, real life, and mathematics	Cultural, community, and personal assets	Mathematics, technology, and art	Mathematics, real-life	Hands-on	Involving parents

We wanted to involve parents by encouraging the girls to explore family stories and by inviting parents to the exhibition. If there is more family involvement, the project is more likely to become everyday practice.

The integration characteristic of the project involves creating a task that does not connect to one specific subject but instead transfers from a mathematics task to an integrated STEM task. Though less focused on mathematics, this provides multiple entry points for students, allowing them to bring their cultural and life experiences to the project. We have not tested this task in GEMS. Laura and I both like the project and see this project as not "too school" but fun, which aligns with the mission of GEMS.

Scaling Task

Since the fall of 2021, Laura has been implementing an engineering-focused STEM curriculum developed by STEM Education Works (https://stemeducationworks.com) in her GEMS club. In the Fall 2021 semester, the girls worked on building clubhouses. I was allowed to observe the GEMS club through a virtual platform. In the first couple weeks, the middle school girls learned the structure of the clubhouse and became familiar with materials and tools. Then, they worked collaboratively to cut the wood and build their own clubhouses. Before proceeding to the building stage, they were introduced to new geometric concepts (e.g., scaling, ratio, views of 3D shapes) to prepare them for understanding the terms later in the building process. This was also an opportunity for the girls to learn geometric concepts and see mathematics through an engineering project.

Identification

In one session, Laura introduced blueprints and scale models to the girls. To support the girls' experiences with scaling, they were asked to create a scale model using a given scale. Laura provided the girls with some original LEGO designs and asked them to take the same design and re-create the design by scaling by two. Figure 5 shows the original designs and correct models from the curriculum, and two samples of students' work.

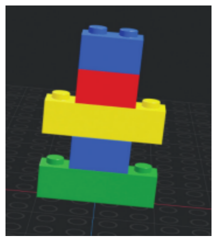
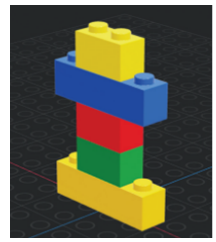
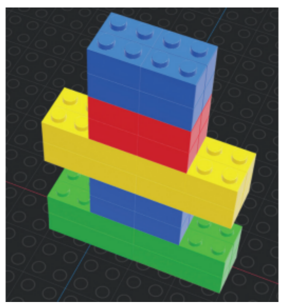
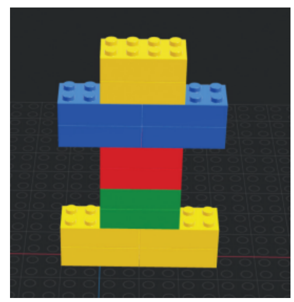
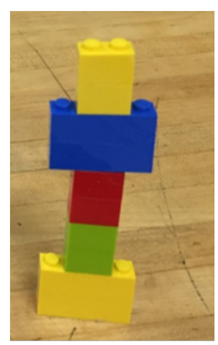
Initial Design		
Correct Model		
Sample of Student Work		

Figure 5. Scaling Task

Laura gave the girls brief instructions about the task. Afterward, the girls worked individually to recreate the designs, then they showed their work. Laura was surprised by the girls' work because they created very different designs, even though they followed the same original design, and many of them differed from the instruction book. Realizing something was wrong, Laura did not reach an immediate conclusion or further explain the task. She said to girls, "It's not necessarily which one is right or wrong; they are just different."

After the session, Laura called me and told me that she realized something was wrong but could not figure out the problem. During the phone call, I could sense Laura was upset that she could not figure out what the problem was in the moment. I said the confusion might be because the girls did not understand scaling toward which direction(s) regarding height, width, or length. Via phone, we determined that scaling means expanding the design in every dimension (i.e., width, length, and height). The confusion arose because we did not introduce this concept clearly before the task. Some girls expanded the models in one dimension, and some girls expanded the models in two dimensions (see Figure 5).

Laura sent an email to the curriculum designer:

> I am doing this with my middle school GEMS club, and we did the Lego activity today. I am sure I did something wrong in explaining scale. You can see that one set of girls did the building like the picture and one set just made the LEGO figure two times taller. I was flabbergasted.
>
> The pictures of the completed figures that are scaled look like the builders made it 4 times bigger. Do I no longer deserve my master's degree? (Laura, Email, September 2021)

Later, the designer confirmed that the concept of scale means expanding the figure in every dimension. Laura forwarded the curriculum designer's response to me. I replied: "Scaling in real life refers to enlarge/reduce in each dimension. We should define scaling and model two blocks before asking girls to build the models."

Coordination

Later, Laura and I had a chance to further discuss this task. We interpreted the students' work as they understood scaling by a factor of two as using double blocks to make it bigger. Because the shapes are three-dimensional, they are not sure in which way to expand the original design. The confusion resulted from the lack of precise understanding of scale. Rather than using double the number of blocks, scaling a design by a factor of two means the redesign needs eight times more blocks than the original design, which caused the conflict. Potentially, students could question why they would use *eight* times more blocks. This presents a learning opportunity to connect different mathematical representations (i.e., geometric and arithmetic) (see Figure 6).

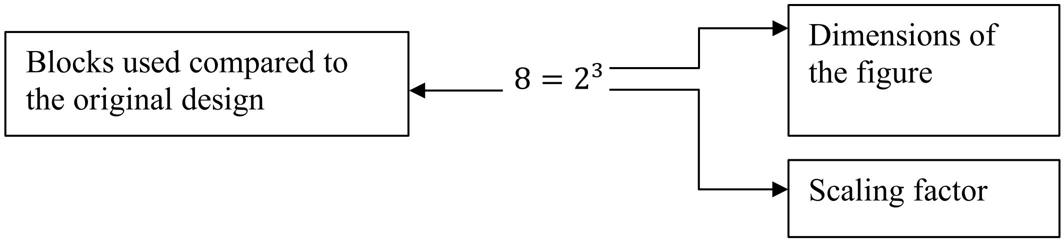


Figure 6. Arithmetic Representation of the Scaling Task

We believe that, to reinforce their understanding, the girls should be given extension opportunity to explore the models and make connection between spatial representations and numbers.

Reflection

My reflection on this task was:

> The mathematical concept conflicts with common sense that scaling by two means to double a shape. The confusion results in a lack of understanding of scaling as a mathematics concept. We use scaling in our daily life, which does not require rigor, while in mathematics, concepts need to be precise to describe the world. (Lili, Reflection, October 2021)

I had noticed that Laura thinks I am more of an authority in mathematics, and she often does not feel comfortable discussing mathematics with me. I am conscious of my privilege in mathematics; meanwhile, I value Laura's practical knowledge by inviting her to further discuss the task. By crossing the boundary between a mathematics person and a non-mathematics person, I am negotiating with my inner voice about what mathematics is. Is it a discipline that highlights precision? Or is it a human practice that prioritizes joy and satisfaction? Or is there a way to join them together?

Currently, Laura feels more comfortable sharing her confusion and questions with me through discussions about a specific issue. She sees me as a "math person" and seeks a solution from me. I told her that I did not immediately notice what happened when I observed the girls' work. I made sense of the problem during our phone call. Thus, there was nothing special about my thinking. I showed this task to my mathematics education colleagues, who also were perplexed at the beginning. Laura said, "I know I need to learn this from you."

Transformation

Laura and I both recognize the importance of the concept of scale/scaling. This observation reminded us to investigate how scale/scaling is described in mathematics curriculum. Using a word search, I found throughout Common Core State Standards for Mathematics (CCSSM) (CCSSI, 2010) that the word *scale* appears 19 times, *scaled* appears three times, and *scaling* appears twice. These instances are distributed across multiple topics: (a) measurement; (b) multiplication; (c) drawing; and (d) transformation.

Although the word *scale* covers multiple mathematics topics in school curriculum, there is no clear description of scale or scaling. Following the first mention of *scale* in Grade 2, the standards in CCSSM expect students to be able to draw the concept from their life experiences. We can infer that the conflict Laura and I experienced is likely grounded in the fact that concept of scaling is not well defined. It is also problematic that, without describing scaling itself, the standards in CCSSM expect students to interpret multiplication as scaling.

We have identified this issue from our informal learning experience in GEMS, which connects to the mathematics concept in the engineering project. Like the Quilt Task, the integration characteristic of the scaling task directed me to position mathematics in a STEM context. I have an opportunity to explore mathematical concepts in an engineering project. Given the importance of scale in mathematics and other disciplines (e.g., engineering), Laura and I recommend that the mathematics curriculum should provide a rich description of scale/scaling.

Chapter Summary

In this chapter, I explored Laura's and my past experiences with and viewpoints of mathematics, as well as our evolving experiences and perceptions of mathematics during our collaboration. Situated in the GEMS context, I discussed the role of mathematics in GEMS. I also used two mathematics tasks, which evolved in the collaboration, to illustrate our boundary crossing experiences between mathematics and STEM and everyday practice. The four learning mechanisms of boundary crossing process (i.e., identification, coordination, reflection, and transformation) provided an analytic lens to unpack the boundary crossing accomplishments and challenges. Specifically, the boundary crossing experiences generated new practices that enriched curriculum in informal practice and contribute to scholarship in mathematics education. In the last chapter, “Discussion and Conclusion,” I respond research questions and discuss the findings on girls-only, informal STEM learning environments. I revisit the theoretical framework and methodology and reflect on the theoretical supports for the study. I also discuss the contributions and implications of the study and provide the conclusion for this dissertation.

CHAPTER 7. DISCUSSION AND CONCLUSION

In the previous three chapters, I presented findings from this inquiry. Based on these findings, in this chapter I first revisit and provide holistic answers to my research questions. Next, I connect these findings with extant literature and discuss girls-only, informal STEM learning environments, like GEMS, and focus on the opportunities and challenges in promoting women's participation in STEM. Then, I reflect on the theoretical framework and methodology and discuss the theoretical supports for the study. This is followed by a discussion of the contributions and implications of the study for the fields of mathematics education and informal mathematics/STEM education. Finally, I discuss future directions for research and practice and provide the conclusion for this dissertation.

Discussion of the Research Questions

In this study, I explored the following research questions:

1. How do two female educators' experiences and histories inform their personal practical knowledge to empower girls and women in mathematics/STEM?
2. What are the boundary crossing experiences of two female educators engaging in Girls Excelling in Math and Science (GEMS) collaboration?
3. What role, if any, do two female educators' mathematical experiences play in providing high-quality informal STEM environments for girls?

In this section, I provide a holistic discussion in each subsection to respond to the three research questions respectively. First, I discuss how our personal narratives shape and inform our personal practical knowledge empowering girls in mathematics. Then, I share the intrapersonal and interpersonal boundary crossing experiences as Laura and I engaged in GEMS collaboration. Last, I discuss the contributions of these boundary crossing mathematics experiences to providing high-quality informal STEM environments.

Narratives of Experiences as Empowering Practical Knowledge

Educators' personal narratives are constructed within social contexts that mediate self and educational contexts and inform professional identity. As educators, we need to understand where we came from, who we are becoming in the present, and how past and present experiences shape our future lives. Specifically, this study explored educators' personal and professional narratives and identities as sources shaping our teacher knowledge in informal learning environments (Connelly & Clandinin, 1988).

I started this inquiry wondering why Laura has devoted 28 years to advocating for girls in STEM. Through the investigation, I traced Laura's life histories and explored her multiple personal roles (i.e., daughter, wife, mother) and her professional roles (i.e., teacher, educator) to help understand why she has dedicated her efforts to developing GEMS. Retelling her personal and professional narratives allowed Laura to reconstruct the stories of her experiences and rediscover how these experiences informed her knowledge as a female informal educator.

My personal narratives resonate with Laura's experiences, revealing how my personal histories shape my teacher knowledge in multiple contexts. Exploring personal narratives with a focus on being women and educators helps provide an alternative way of doing research in mathematics/STEM education. Through storytelling, Laura and I developed a mutual understanding of each other's narrative trajectories of becoming educators, which laid the foundation for our reciprocal collaboration. Through understanding ourselves, we can better navigate our role in formal education and facilitate learning by transferring our experiences into mathematics-related STEM curricula.

As we conceptualize mathematics as a social and cultural practice, female educators' mathematics experiences should not be separated from their experiences of being women. Being a woman is a social construction that interacts with other social practices, such as learning and teaching mathematics. In this exploration, being a woman impacted Laura's experiences as she tries to balance mathematics performance and understanding of mathematics; it also informs her experiences as a mother of a mathematics learner, concerned about her daughter's attitudes in mathematics. In particular, being a woman motivated her to start GEMS for girls to assist them in learning mathematics/STEM. As for my experiences, I found my strength in mathematics, which helped me overcome the stigma of being a woman in mathematics and further empowered me in the field of mathematics. Experiences of being a woman and experiences in mathematics are inseparable and inform one another.

Informal educators have more room than classroom teachers to bring personal practical knowledge into informal learning environments. Different from knowledge in the traditional sense, personal practical knowledge encompasses what is meaningful to the educator, not necessarily the skills the educator possesses. For instance, Laura's negative experiences with spatial skills informed her practical knowledge as she develops appropriate curricula to support girls' spatial understanding. Laura brings her knowledge to the GEMS team, which then simulates design and research regarding spatial curricula. Accordingly, these new practices expand Laura's experiences and extend her personal practical knowledge.

In addition, the narratives seek to present my own experiential histories and boundary crossing experiences as one example of exploring the transformative identity of being an educator. The journey of this inquiry involves a process of developing a new way of thinking and understanding mathematics. In so doing, this study invites educators (informal and formal) to reflect on their own personal histories and boundary crossing lives as a source of knowledge that empowers alternative approaches in teaching and research. Therefore, this study provides a way of understanding educators' personal and professional narratives that contribute to the body of literature on teacher practical knowledge in mathematics, which is rarely explored in mathematics education.

Intrapersonal and Interpersonal Boundary Crossing through GEMS

I heard about Laura in the summer of 2018 when I returned to the United States after reuniting briefly with my family in China. I had just completed my first year in the Ph.D. program and had transitioned from a classroom teacher to a Ph.D. student. Working in the U.S., I had to cross cultural and linguistic boundaries to participate in the U.S mathematics education community. I struggled a great deal to cross these boundaries and settle into the community. Since my first semester, I have been teaching mathematics methods courses for secondary mathematics pre-service teachers and have worked with the secondary mathematics team to prepare the lessons. In the courses, we include various topics related to mathematics teaching and learning, such as equity in mathematics education, mathematical identity, and mathematics knowledge in teaching. Though these topics were not unfamiliar to me as an experienced teacher, they were peripheral in my teaching because I mainly focused on the mathematics itself. I discovered that shifting this peripheral knowledge to a central focus became a source of tension for me in navigating my role in the community. I felt that my knowledge in mathematics was not valued and found it difficult to convey my knowledge when teaching mathematics methods courses.

The tension continued as I returned to my school in China and worked with my former colleagues. In a reflection, I wrote:

> I went back to China in the summer and worked with my previous colleagues for 5 weeks. On the one hand, I realized I have changed a lot regarding mathematics teaching. For instance, I never thought there are other ways to teach mathematics than giving a lecture to explain concepts. On the other hand, I am perplexed when I talked with my colleagues because they reminded me of the reality. I know the reality means a centralized education system and pressures to improve students' test performances. My primary research interest is about professional development for in-service teachers. Now I am doubting that, since I am not sure the research will be meaningful for teachers' empirical practices. (Lili, Reflection, August 2018)

I heard about Laura from my advisor, Dr. Newton, who told me Laura's story and said, "You have a daughter, and you are concerned about her development. You may have interest working with Laura in this program." Laura's story about starting a program for her daughter piqued my curiosity. I wondered how a woman, as a mother, could possibly make such a contribution to education. I said, "Yes, I would love to meet this mom and learn more about her experience." When I met Laura and joined GEMS, I was feeling uncertainty about what research direction to take.

Likewise, Laura also felt uncertainty at that time. She had decided to bequeath GEMS to Purdue University but did not know what it would look like. Together we embarked on this experimental journey—finding a home for GEMS and navigating a direction for my research. Working together in GEMS, both Laura and I have engaged in interpersonal level and intrapersonal level boundary crossing experiences (Akkerman & Bruining, 2016). Laura comes from an informal STEM community, whereas I am from a mathematics education community. Our collaboration allows us to bring our own past experiences and knowledge and engage in a new practice, which results in new experiences and knowledge being generated in the collaboration.

As an intrapersonal level, boundary crossing experience, the emerging scaling task described in Chapter 6 is an example that helps explain the challenge and significance of crossing boundaries between mathematics and STEM, as well as between informal mathematics and school mathematics. In the identification process, Laura recognized that the concept of scaling had not been established in the engineering project. I identified that scaling by a factor of two in engineering or mathematics is different from double/twice an object in everyday language. In the coordination process, Laura began to realize the preciseness of mathematics concepts in resolving confusion in engineering. I started to explore the connection between the scaling concept in mathematics and the three-dimensional model task in the engineering project. In the reflection process, Laura recognized that students should understand concepts first before moving to the actual building stage. I decided to explore how the mathematics curriculum (CCSSI, 2010) presents the concept of scaling and whether the mathematics curriculum supports cross-disciplinary connections. In the transformation process, Laura felt more comfortable bringing mathematics into GEMS by explaining scaling as a mathematics concept to the girls. I developed a connected perspective between mathematics and other STEM disciplines (e.g., engineering).

In interpersonal level boundary crossing, as I illustrated with the emerging quilt task in Chapter 6, Laura and I continued to define our own expertise, share information, value each other's perspectives, and collaborate on a shared problem. In the process, our common identity as an informal educator was reinforced. In the identification process, Laura recognized my expertise in mathematics, while I accepted Laura's core motive was to promote learning in a way that was different from the school setting. In the coordination process, Laura and I exchanged information and shared our cooperative work. Laura sought out relevant information that connected mathematics and quilting and shared it with me, while I tried to make mathematics interesting and fun. In the reflection process, Laura conceptualized mathematics in a broad way that led to acknowledging mathematics in the task. I adopted Laura's perspective in informal learning to cross the border of mathematics and connect it with other disciplines. Finally, in the transformation process, Laura and I collaboratively created the quilt project that highlights mathematics elements that could be used in informal learning environments. This interpersonal level boundary crossing experience strengthened our informal educator identity by recognizing learning opportunities in everyday practices. In addition, we both developed a more open and connected attitude toward mathematics in the interpersonal level boundary crossing process.

Contributions to High-Quality Informal STEM Learning

The interpersonal and intrapersonal boundary crossing experiences I described above serve as sources that have broadened our understanding of formal and informal mathematics. Laura and I joined the GEMS team with different roles. Laura attends the team meetings as the founder of GEMS and an experienced leader in informal STEM learning environments. She rarely sees herself as a researcher, though she has done action research with the original GEMS girls. I participate in the GEMS team as an experienced mathematics teacher and novice researcher. Laura and I accept our different roles and identities as *legitimating coexistence* (Akkerman & Bakker, 2011).

During our interactions, Laura plays the role of a mentor to support me in becoming familiar with GEMS as well as informal STEM learning. At the same time, she is conscious of my role as a researcher, saying, "Tell me how I can help and what do you want to know from me." When we work on mathematics-related activities, Laura refers to me as "you math person." I see Laura as an expert in informal learning and as a mentor who provides guidance and motivates me to learn about and participate in informal education. At times we encounter the boundaries between our different roles, and small tensions emerge when we discuss GEMS activities. The mathematics educator identity I hold seeks to determine whether an activity can provide a learning opportunity for students to recognize mathematics. However, Laura, as an informal educator, is concerned about whether an activity will engage the girls; she also feels that mathematics does not always have to be highlighted unless it can point to future STEM career paths. The differences in our identities lead to a negotiation about the focus of mathematics in GEMS.

When we get into the coordination process, with different perspectives, we work toward *communicative connection* (Akkerman & Bakker, 2011). Laura represents the informal education community, while I represent the mathematics education community. Mathematics-related activities that we work on serve as boundary objects that mediate tensions caused by our different identities. This boundary object allows for diverse practices and ensures cooperation in the absence of consensus (Akkerman & Bakker, 2011). The boundary object lends itself to multiple practices and meanings (Wenger, 1998). For the mathematics education community, mathematics-related STEM activities focus on mathematics; in turn, mathematics concepts can be developed through STEM integration (Fizallen, 2015; Shaughnessy, 2013). For the informal STEM education community, mathematics-related STEM activities inspire students who potentially participate in STEM fields (Beswick & Fraser, 2019). I am concerned about what role mathematics plays in integrated STEM activities, whereas Laura cares more about whether the activities are fun and target STEM careers. Though we often prefer different emphases in the activities, when Laura and I engage in developing mathematics-related tasks, we work toward reframing mathematics in ways that serve both sets of goals.

During the reflection on our boundary crossing experiences, Laura and I become reflective about our own perspectives at the boundary. Laura explicitly makes her perspectives (Boland &

Tenkasi, 1995) on GEMS which is an informal STEM learning environment for girls to ensure that a girl sees herself as a change agent and as a person who makes a difference. Laura has gained a new perspective on mathematics and embraces the idea that promoting mathematics in GEMS can support its mission. I have my own perspective—that understanding my position at the boundary enriches my current identities as researcher and teacher educator and forges a new, evolving identity as an informal educator (Akkerman & Bruining, 2016).

The new practice of developing spatial curriculum is created through transformation, which "entail[s] the emergence of new in-between practices" (Akkerman & Bakker, 2011, p. 150). Our various identities (e.g., mathematics teacher, informal educator, researcher) inform one another throughout the transformation. The new practice of developing spatial curriculum does not necessarily change our established practices and current identities, but rather invites cooperation between practices and moving beyond the boundaries Developing spatial curriculum is new and unfamiliar to both Laura and me which requires us to enter new roles as curriculum developers. I have developed the research agenda on examining spatial concepts in the curriculum, while Laura introduces mathematics as a natural tendency in GEMS through spatial activities.

For both of us, participating in GEMS has taken the form of crossing disciplinary boundaries (i.e., mathematics and STEM), as well as crossing the boundary between mathematics education and informal education. The boundaries set up contexts for practice, which carry learning potential for us (Akkerman & Bakker, 2011). Learning alters and reinforces our identities as informal educators (Lave & Wenger, 1991). The shared identity, knowledge, and interpersonal relations in the practice encourage our collaboration, which creates high-quality informal STEM learning environments for girls by humanizing mathematics curriculum. More details about humanizing and reframing mathematics are provided later in the Contribution section.

Discussion of Girls-Only Informal STEM Learning Environments

Girls-only and informal learning are two key characteristics of GEMS. Based on the findings of this study, in this section I connect with previous literature and discuss girls-only informal STEM learning environments. First, I discuss promises and challenges of constructing feminist learning environments. Second, building on the exploration of the long-term standing informal learning environment—GEMS, I extend the literature on informal education by discussing aspects of informal STEM learning and informal educators, and I present a challenge for informal STEM education.

Constructing Feminist Learning Environments

School mathematics, especially traditional approaches in mathematics, often denies students access to a depth of understanding (Boaler, 1997). Boaler posited that boys might be more adaptive to traditional learning environments, while girls' preference for understanding probably works to their disadvantage. If mathematics teaching alienates girls, then negative attitudes are inevitable, regardless of their performance. This might explain Laura's confusion about Janet's attitude toward mathematics—her mathematics performance in school was good, but she did not have confidence in mathematics. Laura herself also had similar experiences; she did not understand "why you have to prove something when it's very clear that that's a 30-degree angle." Understanding concepts contributes to girls' development of long-term interests in mathematics. Girls might not be convinced to do mathematics in a certain way without understanding why the approach works. An OGG, Stella, communicated her trauma in memorizing times tables in the classroom, but she enjoyed how GEMS helped with understanding times tables.

Promises

Boaler (2002) proposed that the disaffection of girls in mathematics might be considered a response to particular teaching environments. Furthermore, Boaler argued that without analysis of the learning environment, research on gender differences in mathematics might mislead and co-produce gender inequities. The rise of feminist pedagogies in mathematics education in response to gender-related preferences promotes girls' learning, which I discuss in the next subsection.

A feminist learning environment uses inclusive and non-discriminative pedagogies to value girls' voices, improve equity for women, and promote connected knowing (Anderson, 2005). For Laura and me, GEMS is a promising feminist learning environment with a mission to promote girls' participation in STEM fields. GEMS is a learning environment only for girls and those who self-identify as girls. Laura believes that girls need such a space to redeem their lack of opportunity to experience STEM in school. Teaching and learning are redefined in GEMS. The leader is not necessarily the most knowledgeable person; instead, she is part of the learning group and is willing to explore new knowledge with the girls. Thus, the leader is not an authority but a facilitator, organizer, and participant in learning; she listens to the voices of girls carefully, responsively, and actively (Jacobs, 2010).

Although Laura rarely designs activities for GEMS, she diligently looks for resources by searching websites, asking people in STEM areas, and attending relevant STEM conferences. She attempts the activities herself before she introduces them to the GEMS community. In addition, she is open to emerging questions and allows the girls to express their thoughts and ideas. She also straightforwardly responds to the girls' questions and concerns to increase their understanding (Boaler, 1997). In doing so, she creates a climate for caring and promotes connected knowing based on care (Chapman,1993).

Feminist learning environments promote connected learning that builds on trusting personal experiences and interpersonal communication (Belenky et al., 1986). GEMS leaders create such an environment by valuing collaboration and focusing less on competition (Becker, 1995; Chapman, 1993). In GEMS, girls are working in small groups, sharing experiences, and discussing their ideas. Collaborative learning in GEMS also is confirmed in the OGGs' description. Leaders trust the girls' thinking and encourage alternative methods. Instead of seeking confirmation about right or wrong, the girls gain confidence from exploring multiple approaches to solve a problem (Anderson, 2005). Thus, a feminist mathematics learning environment provides a new way for girls to understand mathematics.

The desired way of learning in a feminist learning environment calls for reframing curriculum to create a problem context that is directly or indirectly relevant to the personal experiences of students (Chapman, 1993). Students are then engaged in interactions with peers, focusing on the learning process rather than on results, which breaks the dichotomy of mathematics (i.e., right or wrong) and opens mathematics to a broader audience (Burton, 1995).

Challenges

Collaborative learning could be a challenge in feminist learning environment. As Chapman (1993) cautioned, collaborative learning is not just putting students in small groups to share mathematical ideas, and teachers should be aware of the unique features of connected knowing and be able to facilitate connected knowers. Tasks and activities should be designed to prompt collective effort. One of the original GEMS girls, Stella, commented that activities in GEMS promoted collaboration:

> The nice thing about GEMS was that all of us were at a baseline learning about something really new. None of us really understood it and nobody was good at any of it yet. It was all new. So, because of that, you learn a little bit more about group dynamics. [The leader] was really great about just promoting what people are good at and figuring out what people are good at. (Stella, Interview, August 2020)

Therefore, in addition to implementing collaborative tasks, collaborative learning requires that informal educators break with traditional ways of learning that focus on individual development and instead develop a collaborative learning mindset.

Another challenge for creating feminist learning environments is how to communicate with girls about such environments (e.g., women in mathematics/STEM). Some researchers believe that the ways teachers and parents communicate with girls might continue the stereotypes (e.g., Hill et al., 2010). Laura does not recommend GEMS leaders sending the message of fewer women participating in STEM fields to girls. In the GEMS handbook, she explained:

> If we told our GEMS members that we started the club because there are fewer girls and women in STEM and because girls in general seem to stop wanting to go into these fields, we are subliminally suggesting to them that this will happen to them. (GEMS Handbook, 2019, p. 17)

She continued:

> How much better is it to say, "We are inviting you to join this fabulous club where you can do all these cool math, science, and engineering things and meet people who do them for a living"? What girl wouldn't like to be in that club? (GEMS Handbook, 2019, p. 17)

Educators' gendered views of mathematics/STEM might be an obstacle for girls further pursuing STEM fields (Jaremus et al., 2020). Thus, support is needed for informal educators to develop the conciseness of potential stereotypes and to be mindful about how they communicate with girls. Without the awareness of potential negative messages, informal educators risk continuing the stereotypes they seek to change.

Supporting Informal STEM Learning

As a designed informal STEM learning environment (NRC, 2010), the mission of GEM is to provide alternative learning opportunities outside of school for girls. Informal learning allows space for emerging questions that may deviate from the prepared topics. In informal learning contexts, mathematics concepts are embedded in authentic contexts, thus students perceive them differently from formal settings (Civil, 2007).

Promises

Informal learning mediates the traditional image of mathematics by valuing individual experiences and subjective knowledge (Zhou et al., 2021). The OGGs recalled that they worked on experiments and activities based on their personal interests (Copper, 2010). Though only a few of them could name such activities, their shared memories revealed that their experiences in GEMS were fun and engaging.

As an emerging field of learning, informal mathematics learning is predominantly unstructured and experiential, with the potential to disseminate alternative images about the nature of mathematics (i.e., what mathematics is and how mathematics is learned) (Burton, 1995). The informal contexts allow greater flexibility to use alternative approaches to solve problems. Rather than completing the problems quickly and correctly, students engage in the learning process and develop persistence in using creative and diverse methods to tackle the activities (Nemirovsky et al., 2017).

Though the boundary between designed informal learning and school learning is, at times, blurry, some designed informal learning like GEMS intentionally differs from school formal learning. Compared with a school's highly structured schedule, GEMS is loosely structured. Laura describes GEMS as "not like school." In school mathematics learning, students are taught to use certain problem-solving strategies. In GEMS, mathematics is unintentionally incorporated into activities. Students are not restricted to using strategies learned in the mathematics classroom; rather, they are encouraged to use creative approaches.

In school, mathematics is not a means but the end in itself, whereas in GEMS, students may not even be aware of doing mathematics. Instead of pointing out the mathematics during the process, Laura prefers to guide students at the end of the activities to reflect on what mathematics they used and how mathematics can support future careers. Students' perceptions of mathematics learned in school are greatly shaped by their school learning experiences (Sam & Ernest, 2000). Calling attention to mathematics in informal learning environments could trigger students' emotionality (like or dislike) of mathematics (Hottinger, 2016). It could possibly diminish students' passion for exploring the activities. In particular, students who are not confident in mathematics might experience anxiety when dealing with mathematics even in informal settings. Not explaining mathematics in an informal learning environment leaves space for redefining mathematics and changing students' traditional perception of mathematics, thus opening a new way of thinking about what mathematics is and how to learn and use it.

Challenges

One challenge of crossing the boundary between informal learning and school learning is increasing the permeability of the nature of mathematics from informal learning environments to the school setting. Informal STEM learning environments provide meaningful and personal activities that increase students' interest. However, having fun is more closely related to temporal interest and weakly related to learning and identity (Morris et al., 2019). For a girl to engage in future STEM careers, she needs to continually and simultaneously develop her STEM interests and solid content knowledge (Morris et al., 2019). We learned from exploring the OGGs' experiences that, although informal learning in GEMS mediated their traditional views of mathematics and provided opportunities for them to experience alternative ways of conceptualizing mathematics, their image of mathematics was still strongly associated with school mathematics (Zhou et al., 2021).

The boundary between school and an informal learning environment often results in discontinuities as individuals engage in the two practices (Akkerman & Bakker, 2011). Maintaining the situational interest developed in an informal learning environment and transferring that interest to school learning requires students to develop substantial knowledge and a positive identity. Nevertheless, most informal learning programs are short term. It is challenging to maintain long-term interest. Although some longitudinal studies have shown that these programs have long-term impacts (e.g., McCreedy et al., 2013), a possible explanation is that girls who participated in these programs were more likely to be motivated or may have already been motivated (Chittum et al., 2017).

Another challenge is lack of professional support for the informal educator. Informal educators come from a variety of educational and professional backgrounds. In GEMS, some leaders are K-12 classroom teachers, like Ms. Cooper; some are parents like Danae; Laura was a preschool special education teacher. An informal educator may not be certified, or they may lack the pedagogical skills to motivate students (Ennes et al., 2020). On the other hand, the nature of integration in informal environments might be challenging for an informal educator who specializes in a particular area. Teaching in informal environments could even be challenging for an informal educator who is a classroom teacher. For instance, when I led the Paper Airplane Club and the Modeling Club, I felt I was teaching out of my field (Luft, 2020).

In addition, the flexibility of informal environments opens the possibility for unpredictable questions and directions of the discussions, which can also be challenging for informal educators who usually do not follow established curriculum or well-developed approaches. As a result, they may become frustrated with what they perceive to be insufficient knowledge and skills to guide students (Allen & Crowley, 2014; Jeffs & Smith, 2021). For instance, Laura did not emphasize mathematics activities in the GEMS club because she felt she could not create activities that were engaging. Collaborating with the mathematics education community and bringing mathematics education researchers' expertise into GEMS improved the situation of inadequate mathematics elements in GEMS.

In this narrative inquiry exploration, Laura and I share a common identity as informal educators. Laura self-identifies as an informal educator. Through collaboration, Laura has developed a broader view of mathematics and now feels more comfortable bringing mathematics into GEMS. I have grown from an outsider to a peripheral participant of the informal learning community (Lave & Wenger, 1991) and eventually I became comfortable calling myself an informal educator. Through exploring Laura's experience and examining my own development as an informal educator, I experience the lives of informal educators and make meaning of informal educators' experiences. In the meantime, my experience of becoming an informal educator through building a cross-disciplinary collaboration with Laura helps to illuminate what is needed to provide professional support for informal educators.

Discussion of Theoretical Framework and Methodology

In this section, I reflect on the theoretical framework and methodology used in this study. First, I discuss the boundary crossing perspective that served as both an analytic and theoretical perspective. The boundary crossing perspective has expanded my understanding of experiences as Laura, and I have crossed multiple boundaries and engaged in GEMS collaboration. Second, I discuss feminist theory, which contributed a theoretical perspective to bring women's voices into the center of the inquiry. Drawing on feminist theory, I value women's professional and personal experiences as a mode of knowing that represents alternative knowledge in mathematics education. Finally, I discuss narrative inquiry as another theoretical support that is compatible with feminist theory and boundary crossing perspective. More than a methodology, narrative inquiry highlights educators' personal and professional experiences as practical knowledge, which reinforces my theoretical perspective on valuing experiences.

Boundary Crossing Perspective Expands Understanding of Experiences

Building cross-disciplinary collaborations potentially create mutually beneficial learning opportunities for educators. Crossing multiple boundaries (e.g., disciplinary, cultural), I continued to work with GEMS, including meeting with GEMS leaders, conducting research with the original GEMS girls, developing GEMS activities, and leading my own GEMS club. The progress was imperceptible, but at some point, I realized that I was a part of the informal education community. I began to use GEMS as a context to discuss informal education and even to help newcomers make sense of learning in GEMS.

The four learning mechanisms at the boundaries: identification, coordination, reflection, and transformation, guide and facilitate my understanding of the in-between situation and the outsider within status and support me to reconstruct identities to understand the past experiences and inform future practice. Through this process, I have forged a new identity as an informal educator and have created a comfortable space for multiple identities. My new identity does not lessen my identity as a mathematics teacher and mathematics educator; rather, these multiple identities have informed one another and have enabled a source of knowledge that reinforces my role as an educator in both formal and informal environments. As I became more comfortable with my new identity as an informal educator, I considered how I could draw on my strength in mathematics to further GEMS practice. To better understand the role of mathematics in integrated STEM, I have taken two integrated STEM courses and had opportunities to work with other disciplinary experts and to hear their views of mathematics.

Boundary crossing perspective also provides a practical framework to support informal educators to cross professional boundaries. Informal educators are required to have a rich, broad knowledge of general education and be able to consistently cross disciplinary boundaries (Jeffs & Smith, 2021; Nemirovsky, 2017). They need to engage in multiple educational communities to develop content and pedagogical knowledge that helps them lead informal learning. Laura habitually moves in and out of multiple communities of practice and diligently draws information and knowledge from multiple communities. Boundary crossing supports her development of informal educational goals.

The use of boundary objects (i.e., activities/curriculum) can facilitate the collaboration between informal educators and other community members. Participating in collaborative practice offers abundant opportunities for informal educators to navigate and construct their identity as informal educators. In this study, the two emerging mathematics tasks serve as boundary objects that engage us to create bridges between two domains of knowledge. Teachers and educators from the mathematics education community should devote their efforts to engaging in informal education, which will broaden understanding of mathematics and potentially reframe and humanize mathematics.

Feminist Theory Values Women's Experience as a Mode of Knowing

Feminist perspectives serve as a theoretical framework for me to hear an individual's personal and professional experiences as a form of knowing. Feminist perspective trusts knowledge construction based on experiences and allows "subjectivity between knower and the known" (Collin, 2016, p. 310). In previous chapters, I explained Laura's experiences as a woman, teacher, and educator; I carefully listened to Laura's and other leaders' stories and located their voices in the center of the inquiry; I heard how informal educators have developed a social justice mindset and, in particular, how they developed consciousness of women's participation in STEM and society. I value their experiences as a source of knowledge in understanding why we need such a space to increase women's participation in STEM fields.

GEMS leaders are committed to nurturing girls' interest in STEM so they can participate in STEM fields later. In their teaching, parenting, or everyday practice, these leaders recognize girls' lower participation rate in mathematics/STEM. They realize that girls can do anything that boys can do; however, stereotypes and norms are obstacles that hinder girls from entering male-dominated fields. Therefore, GEMS leaders intend to provide girls with additional opportunities or alternative approaches to access mathematics/STEM. The primary reason for leaders to devote their efforts to leading GEMS is to make a difference for girls and increase their interest in mathematics/STEM.

Laura said, “I lost my opportunity. I don’t want girls to lose their opportunities like me.” Laura’s career choice was limited because she was a woman from a low-income family. Drawing from her personal experience and family stories, Laura has learned that women often feel inferior in both society and family. Specifically, women’s sense of inferiority in STEM fields is noteworthy. Given the importance of STEM in leading scientific innovation and economic growth, women’s underrepresentation in STEM exacerbates their systemic oppression (Moss-Racusin et al., 2021). Laura created GEMS as a space to work against the systemic injustice and underlying stereotypes about girls in mathematics/STEM.

When GEMS leader Ms. Cooper was a student, she did not enjoy mathematics and science. Following gender norms, she visualized herself as an English teacher, until she discovered there was an overabundance of English teachers and an insufficient number of mathematics and science teachers. Therefore, she decided to teach mathematics and science, and once she tried it, she found she enjoyed it. Ms. Cooper taught in the classroom for almost 30 years and saw the myth that girls are not capable doing advanced mathematics continue. When her school encouraged teachers to open afterschool clubs, she chose to start a math and science club for girls with Laura. She said, “Girls are gems, bright shiny little stones.” In the first GEMS club, Ms. Cooper brought activities that helped girls to see mathematics and science differently. Another GEMS leader, Danae, had heard from her husband that only a few girls participated in the middle school science bowl. She took action immediately and started her GEMS club at the elementary school to conduct science experiments and activities that engaged girls in science and prepare them for the science bowl. She called her GEMS girls “Rubies” and was confident that they were capable of learning and enjoying science and mathematics.

The personal and professional experiences of Laura and other GEMS leaders as a form of practical knowledge indicate a patriarchal culture in mathematics/STEM education (e.g., Harding, 1991). Feminist perspectives challenge the patriarchal culture of objectivity of mathematics that waits to be discovered and rejected the separation of individual knowledge from the community situation and social contexts (Haraway, 2016). Feminist theory enables a fundamental framework for GEMS that reveals power relations behind women’s experiences (Allen, 2018; Hesse-Biber & Piatelli, 2012). The power relations usually emerge imperceptibly. When a young woman selects a career, the factors she considers might be thought of as individual preference. Nevertheless, the personal decision indeed is based on the individual's experiences and shaped by the individual's social position (Haraway, 2016; Harding, 1991).

Through a feminist lens, I recalled my decision to be a mathematics teacher, which I believed was a perfect fit for me. I realized that I did not have other available options to consider, nor was I even aware of other options. Feminist theory focuses on experiences of women and minority communities associated with their social position that provides a critical perspective to understand women's underrepresentation and oppression in STEM. Feminist theory serves as a theoretical basis to justify informal learning environments like GEMS.

Narrative Inquiry Serves Theoretical Support

In Chapter 3, I explained the use of narrative inquiry methodology for this exploration. Here, I discuss narrative inquiry as another theoretical support that guided my thinking throughout the study and is compatible with boundary crossing perspective and feminist theory. Narrative inquiry facilitates the boundary crossing process by encouraging me, as a researcher, to build a sustained and reciprocal relationship with Laura and settle in the midst of GEMS (Clandinin & Connelly, 2000). As a researcher from an Eastern culture, researching the Western world requires me to understand the perceptions and experiences of individuals in their social and cultural contexts. Engaging in this exploration, I crossed linguistic, cultural, and disciplinary boundaries to develop an understanding of the complexities and fluidity of Laura's experiences and identities. I rejected the traditional researchers' neutral stance and engaged in authentic communication with Laura as a person (Pinnegar & Daynes, 2007). Being a real person somewhat relieves tensions that are created in the *in-between* (He, 2002) spaces as I crossed the multiple boundaries. Typically, the three-dimensional narrative inquiry space allows me to move backward, forward, inward, and outward to explore my own narratives as a source of knowledge.

There is great compatibility between feminist perspective and narrative inquiry. Narrative inquiry and feminist perspective share a focus on the connection between experience and knowledge. Feminist research begins with women's own perspectives and experiences and confronts socially constructed gendered inequalities (Jagger, 2016). Specifically, feminist researchers support that individuals' daily activities and lived experiences structure their understanding of the social world (Hesse-Biber, 2012). Similarly, narrative inquirers value participants' lived experiences and claims that knowledge generates from narrative cognition and social relationships (Bruner, 1986).

With its focus on experiences as a source of knowledge, narrative inquiry provides a methodological approach to investigate women's lived experiences, while feminist perspective offers a theoretical perspective to examine experiences from women's perspective. Further, narrative inquiry emphasizes interactions between researcher and participant and highlights the researcher's experiences (Clandinin & Connelly, 2000). Similarly, feminist perspectives allow reflexivity in identifying the cultural values and interests of the researcher (Harding, 1991). The compatibility of narrative inquiry and feminist perspective allowed me to explore women's experiences as social construction.

Thinking narratively enabled me to bring Laura's stories and my own experiences together in a connected way with a common identity as female educators. Narrative inquiry encourages collaboration between the researcher and the participant, which supports the possibility of boundary crossing exploration. Moreover, narrative inquiry is an empowering approach to value female educators' personal experiences as a mode of knowledge. Through the storytelling of two female educators' personal and professional experiences, I have embraced the understanding that our views of mathematics (i.e., what is mathematics) are changing in day-to-day boundary crossing collaboration.

Contributions

This study is situated in the intersection of mathematics education, informal education, and women's studies, all of which significantly impacted Laura and me, as well as GEMS, where this study took place. A significant effect can also be seen in the mathematics education and informal education fields, which offer possibilities of building boundary crossing collaboration between these two communities. This study bridges a "substantial divide" (Damarin, 2008, p. 101) between women's studies and mathematics education research by highlighting women's experiences in mathematics/STEM.

Reframing and Humanizing Mathematics

Laura and I were born in different times and cultures. Our primary mathematics experiences occurred at school. Laura enjoyed playing cipher games, which differed from school mathematics. I practiced on the abacus and solved puzzles at home, which, to me, did not seem to have a strong connection with school mathematics. In informal teaching, Laura and I both recognize the difficulty for teachers to promote mathematics in informal learning environments. First, the casual aspect of informal learning allows for ambiguity, which does not align with traditional views of mathematics, such as absolute, neutral, formal, and rigorous. Also, since every informal learning environment has its own characteristics and focus, no curriculum exists that is applicable to all informal learning environments. Developing a curriculum that highlights the value of the informal learning environment, like GEMS, takes great effort.

Through this interdisciplinary collaboration, I, as a mathematics education researcher, had opportunities to cross boundaries between mathematics education and informal education; hear mathematics stories from Laura, who does not work in a mathematical field; and collaborate with her to develop a mathematics curriculum. These unique experiences allowed me to reframe and humanize mathematics by viewing the mathematics experience as a human practice. The experiences led to a transformative understanding of mathematics learning that positions learners and teachers as subjects in mathematics, values the experiences they bring to learning, and views both learners and teachers as objects of learning and their development as the goal of learning.

In our collaboration, Laura and I committed to promote humanized mathematics (Wheeler, 1975) that allows GEMS to be a potential site to interrupt traditional views of mathematics. We aim to make mathematics less formal and significantly different from the mathematics that students encounter in school. Specifically, Laura wants to make activities in GEMS ahead of students' grade levels and different from their classroom learning. As Stella, an OGG, recalled, because the curriculum is new to everyone, the assumption—based on classroom experiences—that some are good at mathematics and others might not work well in the GEMS setting. As such, students gain access to mathematics that does not necessarily correlate with their performance in school mathematics. Informal mathematics potentially interrupts the approaches students use in the mathematics classroom and mediates the traditional image of mathematics (Zhou et al., 2021).

When we highlight the fun and interesting components of mathematics in GEMS, we perceive mathematics as more than an individual school discipline; instead, we view its connections with everyday practice. In the quilt project, we intend to reframe and humanize mathematics that embodies the mission of GEMS. Initially, the quilt task we designed was primarily mathematics-focused and assumed a fairly traditional mathematical approach. As we continued conceptualizing mathematics as everyday practice, we incorporated culture, art, and technological elements into the project. We hope that participation in the quilt project can foster students' awareness of self and the world (Gutierrez, 2012). The quilt project allows students to use their cultural and community experiences as a form of knowledge to participate in the project; meanwhile, they understand each other's product as a part of the world (Falkenberg, 2006). Drawing on e expect that the quilt project can serve as a mirror that allows students to value their cultural, community, and everyday experiences, as well as a window that provides opportunity for students to see broad connections between mathematics and other practices.

The way we reframe and humanize mathematics in GEMS is to incorporate students' individual experiences as a mode of knowing in mathematics curriculum. We believe that acknowledging students' everyday practice as a source of knowledge can help students fully realize their potential in mathematics. Specifically, valuing girls' learning traits directly challenges underlying stereotypes of females' inferior mathematics ability embodied in socio-cultural contexts (Lubienski & Ganley, 2017).

Crossing Contexts Research in Mathematics Education

Reframing and humanizing mathematics as normal, ordinary, and relevant to people is an urgent task for the mathematics education community (e.g., Darragh, 2018; Sealey & Noyes, 2010; Stephan et al., 2015). Efforts taken to cross boundaries between informal education and mathematics education can build bridges between in-school and out-of-school mathematics, providing opportunities to reframe and humanize mathematics. The view of mathematics as formal, rigorous, and distant from real life is widely circulated in culture and society, even within the mathematics education community (Sam & Ernest, 2000). Mathematics in the classroom focuses on specific learning objectives to promote students' achievement in mathematics; satisfaction and excitement are not primary goals. Informal learning highlights excitement and engagement, but the mathematics is often hidden (Civil, 2007) and remains "silent" (Shaughnessy, 2013, p. 324) when integrated into STEM activities.

The boundary crossing efforts in this study build a bridge between mathematics practice in school and mathematics practice in informal learning environments. Informal learning environments like GEMS encourage excitement, where having fun is a primary consideration for selecting and implementing activities. Such an emphasis can be potentially disruptive to the seriousness assumed to be essential in learning mathematics. Educators from the mathematics education community support identifying and making mathematics explicit in STEM integration, whereas educators from the informal education community contribute their expertise to bring excitement and engagement to mathematics learning. Learning across boundaries leads to discontinuities (Akkerman & Bruining, 2016), which can be understood as learning mechanisms to generate new practices that create a space for reframing mathematics. Though tensions and struggles are inevitable as we cross boundaries to engage in a new practice, tensions potentially stimulate learning and identity development (He, 2002). Laura and I both have been involved in developing non-traditional mathematics curriculum and have developed a new understanding of mathematics.

This study provides an example of building boundary crossing collaborations between the mathematics education community and other disciplines or communities outside of mathematics education to reconceptualize the understanding of mathematics. As such, the study contributes to changing the traditional image of mathematics by reframing and humanizing mathematics as a normal practice that encourages a wide range of participation in mathematics (Darragh, 2018). In particular, the effort of humanizing mathematics in this study has the potential to inspire historically underserved students in mathematics to actively participate in mathematics/STEM (e.g., Joseph et al., 2019).

Toward a Feminist Perspective in Mathematics Education

As we conceptualize mathematics as a social and cultural practice, female educators' mathematics experiences should not be separated from their experiences of being women. Being a woman is a social construction that interacts with other social practices, like learning and teaching mathematics. In this exploration, I found that being a woman impacted Laura's experiences as she tried to balance her mathematics performance and understanding of mathematics; it also informed her role as a mother of a mathematics learner concerned about her daughter's attitudes in mathematics. In particular, being a woman empowered her to start GEMS for girls to learn mathematics/STEM. In my case, I found my strength in mathematics, which redeemed my oppression as a woman and further empowered me in the field. Experiences of being a woman and experiences in mathematics are inseparable and inform one another.

GEMS is a learning environment that responds to girls' experiences and their mathematics/STEM experiences simultaneously. Ms. Cooper referred to girls as gems; Danae calls her GEMS girls "rubies"; they both value girls being girls. Laura encourages snack time in GEMS, which provides the girls with time to chat. She wrote in the handbook, "Snacks provide a huge social benefit to the meetings. Girls are social beings—they like having time to talk." This study calls for constructing feminist learning environments, which not only confirm girls' and women's knowledge, but also potentially promote feminist knowing of mathematics as "person- and-cultural/social-relatedness" (Burton, 1995, p. 287).

A feminist perspective in mathematics education makes mathematics more accessible and enables changes in mathematics and its teaching, thus potentially improving women's participation in mathematics. A feminist perspective values girls' and women's experiences as the knowledge that contributes to rethinking research and practice in mathematics education. Knowing mathematics is related to the person who is claiming to know and to how that knowledge is being presented. Highlighting girls' and women's experiences in mathematics thus potentially challenges traditional views of mathematics and supports changing the public image of mathematics (Stephan et al., 2015).

Implications

Implications of the findings of this study are based on the narratives of the participants' personal and professional lived experiences of becoming educators in informal STEM learning environments. These implications focus on research and practice in mathematics education and informal mathematics/STEM education. They contribute, therefore, to the extensive literature that discusses the possibility of improving women's participation in mathematics/STEM.

Fostering Interdisciplinary Collaboration in Mathematics Education

Promoting interdisciplinary collaboration facilitates boundary crossing between mathematics education and other practices, particularly other STEM disciplines. To make mathematics explicit and transparent in STEM integration (Shaughnessy, 2013), mathematics educators need to make sense of how to build connections between mathematics and other subjects. Interdisciplinary collaboration supports mathematics educators in transforming the traditional mathematics teaching paradigm to a more connected perspective. As such, rather than function as a gatekeeper of access to further education in STEM fields, mathematics should be understood as a gateway for STEM pipelines.

Mathematics education for the 21st century should be developed within the context of STEM education (Beswick & Fraser, 2019). Interdisciplinary collaboration allows mathematics educators to synthesize knowledge across multiple disciplines and address mathematics through content integration. Cross-disciplinary collaboration exposes mathematics to relevant STEM contexts, such that mathematical ideas are conveyed in a connected way instead of using approaches isolated from real life and other disciplines (English, 2017). Linking mathematics with other STEM disciplines supports students' development in both mathematics and STEM integration, inspiring students to graduate and pursue STEM careers.

Interdisciplinary collaboration requires a broader approach incorporating alternative methods, objectives, and content in solidarity. For researchers in mathematics education who seek a cross-disciplinary view of mathematics, narrative inquiry provides a research methodology as well as a coherent theoretical framework that ensures a scholarly space to capture evolving change and growth of personal practical knowledge. Narrative inquiry is an empowering approach that is grounded in lived experience, offering an experiential and fluid way of understanding. Narrative inquiry invites researchers to start from their own narratives of experience and encourages researchers to develop a relational understanding of participants. Through a narrative inquiry approach, mathematics educators can access the experiences of people from other disciplines, including their feelings and thoughts about mathematics. As such, mathematics educators potentially can take outsiders' perspectives and develop a transformative understanding of mathematics.

Advancing Informal Mathematics/STEM Education

As school mathematics focuses on improving individual students' performance to meet educational standards, informal learning environments can provide experiential space to reframe and humanize mathematics by implementing alternative curricula and instructional approaches.

Mathematics in designed informal environments consists of an accessible curriculum and culturally responsive pedagogy, which satisfies learners in working with mathematics. In addition, the relaxed atmosphere of informal learning environments enables doing mathematics within a "normal and ordinary" (Darragh, 2018) action.

Informal learning environments potentially provide more opportunities for social interaction. Developing a mathematics curriculum for informal learning should take the students' social constructs and cultural contexts into account. For instance, girls-only is a particular characteristic of GEMS. Other informal learning environments might serve learners from specific groups, such as the Girls STEM Institute, which specifically supports Black female students (Morton & Smith-Mutegi, 2018). In addition to recognizing learners' personal and cultural experiences, developing a curriculum should integrate these experiences to make the curriculum relevant to learners' past, current, and future experiences.

With the intention to reframe and humanize mathematics, informal learning allows the opportunity to claim knowledge, that is, to identify whose knowledge is valued. Thus, informal learning environments provide space to address socio-political concerns in mathematics education. In particular, humanizing mathematics in informal learning environments can empower learners from historically underserved communities in mathematics/STEM so their voices are heard, and their experiences are valued (e.g., Gutierrez, 2018).

Future Directions

The findings from the study suggest that future research should use a narrative approach to explore cross-disciplinary collaborations between mathematics education and other practice communities and an intersectional feminist perspective (Collins, 2019) to understand multiple dimensions of inequality that women face in mathematics/STEM. This study also suggests a future direction of building learning communities and creating professional development opportunities for informal educators.

Future Research Directions

In completing this work, I recognized that informal learning environments enable the development and implementation of non-traditional mathematics to empower girls and women in mathematics/STEM. Particularly, for girls from multiple marginalized communities, informal learning environments provide learning opportunities that allow girls to see that career paths in STEM fields are available to them. Informal learning would be a valuable context to foster boundary crossing collaboration to develop and implement humanized mathematics tasks. One recommendation for further research is to implement the two evolving mathematics tasks used in this study in informal learning environments and investigate the interactions between tasks and learners and the pedagogical instruction to promote the interaction. The findings could broaden mathematics understanding and further inform curriculum development in informal learning environments.

In relation to humanizing mathematics, a future investigation could continue cross-disciplinary collaboration with educators from other disciplines to broaden understanding of and humanize mathematics. Specifically, this study provides a reference for building a reciprocal relationship with mutual trust in collaboration and for using a boundary crossing analytic lens to investigate educators' cross-disciplinary experiences. A boundary crossing perspective can be continually used to examine the interpersonal and intrapersonal experiences of working with mathematics. By exploring educators' cross-disciplinary experiences, future research could focus on different voices from outside mathematics education.

Another future direction is to extend the feminist perspective in this study to include intersectional feminist theory (Collins, 2019) to better understand the intersections of girls' and women's experiences in mathematics/STEM by exploring their daily actions and interpersonal interactions. Locating the intersections where women's differences regarding multiple social constructs (e.g., race, gender, class, ethnicity) emerge may provide a way for researchers to explicate different dimensions of inequality women face in mathematics/STEM. Intersectional feminist theory provides a unique lens to understand differential experiences, multiple dimensions of identity, and their relationship to systems of power (Collins, 2019). Specifically, girls and women from multiple marginalized groups encounter gender construction-related inequality in everyday interactions. Using an intersectional feminist lens will empower the diversity of women's voices in mathematics/STEM and, in turn, will potentially broaden their views of mathematics.

Future Practice Direction

Future practical work for mathematics education might be to foster alternative ways of conceptualizing mathematics in informal learning environments regarding curriculum and instructional approaches. Informal educators, like GEMS leaders, voluntarily engage in informal education. However, lack of training and support likely leads them to practice teaching in a way that is similar to classroom teaching. To help reinforce the perception of mathematics gained from informal learning and continue constructing broad views of mathematics, mathematics educators could create a learning community and provide professional development opportunities to support informal educators.

Creating a learning community will provide a social learning space for informal educators to build networks and foster collaboration. As informal educators engage in the community, they participate in the community of practice, which reinforces their identity as informal educators (Lave & Wenger, 1991; Wenger, 1998). Educators from the mathematics education community could contribute their expertise in the areas of mathematics and mathematics education by providing mathematics-related integrated STEM activities and a wide range of mathematical tools. Informal educators bring their varied expertise to the learning community and contribute to further developing the community.

Learning communities by nature will generate professional development opportunities. The various disciplinary, cultural, and educational backgrounds of the members allow them to cross boundaries for collaboration and communication. This boundary-crossing communication could facilitate informal educators' involvement in designing and implementing mathematics-related activities. Therefore, it is possible to push informal educators across the boundaries between traditional mathematics and other disciplines and develop a new understanding of mathematics.

Mathematics teacher educators also benefit from involvement in informal learning environments. They naturally carry their broad views of mathematics developed in informal learning environments into their teaching which supports pre-service teachers and in-service teachers to see mathematics from multiple perspectives, such that pre- and in-service teachers have opportunities to reconceptualize mathematics. As a mathematics teacher educator, myself, my teaching in informal settings informs the ways I communicate with pre- and in-service teachers. I introduced two developing mathematics tasks to my pre-service teachers and invited them to work on and provide feedback about the tasks. I also facilitated discussions with in-service teachers about the issues of girls and women in mathematics/STEM and social justice issues in educational fields. The discussions prompted their awareness of (in)justice that proposed possible interventions against potential injustices in their own teaching.

Conclusion

I began this narrative inquiry by wondering about Laura's experiences with GEMS. I wondered why Laura advocated for girls in STEM, I wondered how GEMS empowers girls in mathematics, and I wondered what mathematics is promoted in GEMS. I had a passion that drove me to walk into the midst of GEMS and work alongside Laura. In the journey of inquiry, I developed a transformative understanding of our common identity as female informal educators. As we collaborated, the inquiry unfolded with the meaning of our experiences, and we continued to address girls' and women's underrepresentation in mathematics/STEM fields. I worked alongside Laura and came to experience her everyday life with GEMS. Throughout the exploration, I reflected on our personal narratives of becoming women, teachers, and educators that shaped our identities and informed our personal practical knowledge. In the midst of GEMS, Laura and I collectively rediscovered and reclaimed the past, present, and future of GEMS as a space empowering girls and women in mathematics/STEM. During the boundary crossing collaboration, our views of mathematics have flowed and changed over time. The two evolving mathematics tasks have illuminated our boundary crossing experiences.

During this inquiry, I listened to Laura's life stories, specifically exploring her experiences with mathematics. As I did so, I also reflected on my own personal history and reconstructed the stories of my past experiences. Through investigating the meaning of the stories, I have learned that our mathematics experiences are interwoven with our gender identity of being girls and women. Thus, women's mathematics/STEM experiences should not be considered in isolation; rather, they interact with the social construction of women. Feminist learning environments where girls' and women's mathematics and non-mathematics are valued can be created to empower girls and women in mathematics/STEM.

The boundary crossing collaboration (e.g., disciplinary, cultural) between Laura and me was experiential and personal. During the collaboration, we developed sharing and reciprocal relationships. I have forged a new identity as an informal educator and see myself continually changing. I also experienced Laura's evolving view of mathematics, and therefore, of GEMS. Our teacher knowledge has flourished from our experiential cross-boundary collaborations. This process of understanding teacher knowledge from experiential experiences enables educators to facilitate the development of students by understanding their personal and cultural experiences.

This narrative inquiry study is situated at the intersection of informal education, mathematics education, and women's studies. Using narrative inquiry methodology, educators explored their personal narratives with a focus on understanding their experiential, personal, and subjective teacher knowledge which provides an alternative way of doing research in mathematics education. Researchers take on both the role of researchers and of participants in collaborative research which narrows the gap between research and practice in mathematics education.

Building boundary crossing collaborations with communities outside mathematics education provides opportunities for expanding views related to the nature of mathematics; in turn, promoting interdisciplinary collaborations within mathematics education facilitates boundary crossing to reconceptualize the nature of mathematics. Furthermore, boundary crossing experiences support mathematics educators to develop a connected perspective between mathematics and other STEM disciplines as well as in real life.

As an emerging area, informal education provides learning opportunities for students who are underrepresented in STEM by taking students' social constructs and cultural contexts into account. Therefore, advancing informal mathematics/STEM education has the potential to contribute to social justice in educational fields. In particular, using the critical lens of feminist theory challenges the patriarchal culture in mathematics education that separates mathematics from individuals' experiences and social contexts and provides opportunities for researchers to conduct studies in mathematics education that value individuals' experiences.

This study can provide illumination for future boundary crossing collaborations. It is my hope that working on GEMS with Laura contributes to the evolving conversation on facilitating boundary crossing collaborations between mathematics education and other educational communities, a conversation about promoting mathematics in informal learning environments, and a conversation about empowering women's voices in mathematics/STEM. It is my hope that narrative inquiry methodology will contribute to a new understanding of educators' experiential and personal practical knowledge in mathematics.

APPENDIX A. INTERVIEW PROTOCOLS

First Interview Protocol

8/16/2019

This is a semi-structured interview with the GEMS founder participant. This interview will achieve the main purpose to understand Laura's experiences as a learner learning mathematics, as a parent of a mathematics learner, and the founder of GEMS, focusing on specific emotional moments that had left a strong imprint on her memories and turning points in her life.

Background Questions:

- Which year did you graduate from Purdue?
- I know your major was special Ed, what led to your decision to study special education and did you have a minor?

Learning experience (emphasis on math)

- I know you are a lifelong learner, looking back over your whole life, could you describe a really powerful learning experience in school or out of school?
- I remember last time we met on the GEMS Day, you said you were very quiet when you were a student, can you tell me about yourself and your experiences as a student?
- Can you tell me about your experience in learning math?
- How did you see math when you were a student?

Professional experience

- How many years you had been working in special Ed? how did your work allow you to start a club?
- How do you connect GEMS and special Ed?
- How do these two professional works inspire each other?

Voice/ The whole Person

- Can you tell me about your life at the time you decided to start the first GEMS Club?
- What was of value to you beyond being a teacher? Were you involved in any social groups?
- How would you describe yourself at that time before you started GEMS club?

- How did you make the decision to start GEMS club? How did it truly happen? What was the turning point? What made you see things differently?
- Can you describe Janet at that time? What did it mean to you to start a club for your daughter at that point?
- In your perspective, what did the club mean to Janet?

Philosophy:

- **Math**
    a. I know GEMS initially focuses on math and science, what are your thoughts on how children learn math?
    b. What did you think of mathematics as a discipline in school?
    c. Could you describe a good math teaching and learning environment? What does it look like?
- **Gender**
    a. What made you notice the issue of gender equity? Was it when you observed Janet during the Open House?
    b. How does gender play a role in your personal life, professional life, and in GEMS?

Conclusion

- Is there anything else you want to add?

Second Interview Protocol

10/7/2019

This is a semi-structured interview with the GEMS founder participant. This is a follow up to the first interview conducted on 08/16/2019. This interview has two main purposes: (1) to clarify any responses from the first interview, (2) to focus on the GEMS trajectory from the first club start up to the current phase at Purdue.

Follow-up questions from the first interview

- In the last interview, you mentioned you were in the gifted class from fifth grade to eighth grade, do you know who was placed in the gifted class and why they were placed in the gifted class?

- When you said you were an angry woman in high school, what did you mean by that? Why were you angry? Do you still feel the same way? What, if anything, has changed?
- When Christy and Janet called you a "*femi"*? What did a "*femi"* mean in society at that time? From your perspective, what attitudes did people hold about feminists and what was the climate related to feminism at that point?
- Last time we talked about your role models; besides your mom, do you have other role models, who are they? Why?
- In the book, "*The All-Wise Being*" you wrote, "there was no Ethan's wife's voice in his whole story," then you made up Flora and gave her voice, what do you want to say through Flora? Who is the prototype of Flora? Can you see yourself in Flora?
- You stated that GEMS is specifically for girls because you wanted girls to have more opportunities to access math and science, and you want GEMS to be different from school. Would I be right in assuming that is your philosophy? How does this philosophy come into play when you apply for grants?
- Last time, you said that you did not do too much math in the early GEMS club, because it was like school. In the grant meeting discussion, we talked about reconceptualizing mathematics in STEM, what are your thoughts on the reconceptualization?

GEMS Trajectory

- Last time, we briefly talked about three components of GEMS:
    - Clinton Global Foundation Initiative, when was that conference, how did the conference impact on GEMS?
    - Toolkit, how did you decide to make a toolkit for GEMS? How long it took you from designed, revised to finally published?
    - Workshops. Was that in 2013? How was that workshop like? How did it impact the trajectory of GEMS?
- Last time, you talked that when the first GEMS club started, STEM even wasn't a word. When did you bring STEM into GEMS?
- What is your favorite activity in GEMS? Why?
- Did you have many opportunities to apply for grants related to GEMS? What types of grants?

- What were your most notable successes in GEMS along the way? What made these achievements so important to you?
- What were your greatest challenges in GEMS along the way? How did you overcome them?
- Looking back, what is outstanding story about GEMS? Do other anecdotes that stand out to you?
- What does learning mean from your perspective, how does GEMS aim to promote learning? What is the difference between learning in GEMS and in the classroom?
- How did spatial skills draw your attention? Tell me more about how spatial skills are connected in GEMS. Did you change your view of geometry, geometry proofs?

Politics

- On the GEMS website, you highlight Title IX. From your perspective, how is it linked to GEMS and what, if any, impact has Title IX had on you or GEMS?
- Do you think the political climate over the past 25 years have had any impact on GEMS? If so, in what ways?

Philosophy

- Why do you exclude male leaders?
- How did you come to the decision to give GEMS to Purdue?

Third Interview Protocol

5/24/2021

This is a semi-structured interview with the GEMS founder, Laura. The interview provides a reflective opportunity for Laura after she has been engaging in Purdue GEMS team for three years. Based on our collaboration of developing GEMS, the interview focuses on three themes: (1) Laura's view of mathematics, (2) informal learning, and (3) women in STEM.

Mathematics

- I remember that you mentioned schools currently do not encourage memorization in mathematics. From your perspective which parts of math need to be memorized?
- Many GEMS activities are not math focused, but a lot of math is included in these activities in an implicit way. You said you didn't do much math in GEMS, but OGGs

recognized math in GEMS. Now that you look back at GEMS activities, how do you see math?

- Most people in our GEMS group are from math ed, in our grant meetings, or research meetings, people talk about mathematics and its learning and teaching, how are their views of math similar or different from yours?
- When the GEMS group talks about emphasizing math in these activities, do you have concerns that girls might lose interest in GEMS or that other STEM disciplines might get less attention?
- Moving forward, what role would you like math to play in GEMS? and why?
- When you think about math, is there anything you want to talk?

Informal Learning

- In your experiences with GEMS, what knowledge, skill, attitude, beliefs or disposition should an informal STEM leader have?
- What content knowledge and subject dispositions are the most important for being an informal STEM leader?
- Besides interventions like GEMS, what would you like to suggest to teachers and to parents for promoting children's learning? what else would you suggest specifically for girls?
- What activities do you do with Alina and Aden when you visit them?
- I remember you were worried that Janet wouldn't do educational activities with Alina. As Alina was doing virtual school last year, how did Janet manage this? Did she ask for help? Or did you give her any suggestions?
- When you think about informal learning, is there anything you want me to know?
- How to create excitement in teaching?

Women in STEM

- Why do you think girls need informal learning STEM spaces like GEMS?
- What makes you think that interventions like GEMS should be available at an early age?
- Why and in what ways do you think participating STEM fields can impact women's lives?
- Did you encourage Janet to pursue a master's degree, or did she make this decision?
    - Why did you think it is important?

    - Why did she think it is important?
- What would you tell women who do not have a clear career goal?
- Last time we talked about that we still live in a male-dominated world, although some women are not aware of this. Do you think promoting women's participation in STEM can be a way to make progress toward empowering women?
- I know you always try to leave comfortable zone to a new field, even join Purdue GEMS team is new for you, what did you learn from that? What is the most challenge (exciting) part for moving into a new field?
- What else you would like to share about women and STEM?

APPENDIX B. GEMS ACTIVITY–FLOATING EGG

Grade level: K-5

Materials:

1. Water
2. Salt
3. Eggs
4. Transparent containers
5. Measuring cups
6. Worksheet
7. Rulers

Key terms: density, dissolve, solubility, ratio

Introduction-Magic Water

- The leader shows girls a cup of water and an egg, and ask girls: if I put this egg into the cup, what will happen?

  - Girls may answer the egg will sink into the bottom of the cup or the egg will float in the water.
  - Ask girls to explain their guesses.
    <u>Value that girls have real life experiences or intuition and listen careful what they say.</u>
  - After girls share their experiences, the leader gently put the egg into the cup. Girls see the egg go to the bottom of the cup.

- Then show another cup of water and call it ***magic water*** and put the egg gently in the cup.
    - Girls will see the egg float in the water.
      The excitement and curiosity are expected
    - Let girls guess what is in the magic water.
    - Show a video about Dead Sea
      https://www.youtube.com/watch?v=UFE4_hCe0OU

Activity-Make Your Egg Float

Girls' activity

- Two girls work together
    - Girls choose their partners or assign partners
- Girls work in groups and make their egg float
    - Record the use of water and salt use, and the height of the egg.

Leader activity

- Distribute material
    - Each group has a see-through container, an egg, water, measuring cups, salt, a ruler, and a worksheet.
- Offer help by answering questions.
- Use the terms density, dissolve, solubility, ratio and explain them to the girls
- Listen to girls' discussion and facilitate discussion

Discussion and Reflection

- Guiding questions
    - What did you find from this experiment?
    - Why can the egg float in the water?
    - What did you learn from this activity?
- Crazy questions
    - If we add more and more salt, can the egg fly in the air?
    - If we do not change the amount of water, how much salt we can add?
    - If we add more and more salt in the water, can rocks float?
    - If you use sugar instead of salt, can the egg float?

Summary

- Use girls' language at the girls' academic level to explain terms and the phenomena.
- Tell girls they will learn more about this in their middle science class.
- What kinds of jobs would need to understand these concepts?

Worksheet

Water (Amount in cup)	Salt (Tablespoons)	Egg (Distance from table in cup)

What did you find from today's activity?

What do you want to know more about?

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